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Chapter 1
Fundamental Electrical Terms

1.1 Introduction

It is essential to understand the fundamental of electrical terms to comprehend any electrical phenomena. This chapter presents these necessary concepts and components as a first stepping stone in understanding any underlying electrical principles. In addition, it introduces the core measuring equipment in the electrical domain along with charge, current, voltage, power, energy, resistance, semiconductor and insulator.

1.2 Charge

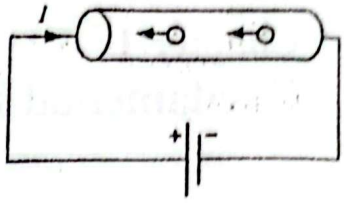
Charge is an electrical property of matter, measured in coulombs (C). The charge in motion represents the current and the stationary charge represents static electricity. There are two types of charge namely the positive charge and the negative charge. Like charges repel each other and opposite charges attract each other. The charge is represented by the letter Q or q and its SI derived unit is coulomb (C). Coulomb is a large unit of the charge and, therefore, generally the smaller units of charge (pC, nC and μC) are used in practice. Note: charge on an electron, which is negative, has the magnitude of 1.602×10^{-19} C. In one Coulomb of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons [1–3].

1.3 Current

The flow of electrons results in current. The direction of the current is opposite to the flow of electrons (negative charges) as shown in Fig. 1.1. The current is represented by the letter I and its SI unit is ampere (A) in honour of French Mathematician and Physicist Andre-Marie Ampere (1775–1836).

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Fig. 1.1 Direction of current



In general, the current is defined as the total charge Q transferred in time t and it is expressed as,

$$I = \frac{Q}{t} \tag{1.1}$$

The rate of change of charge transferred at any particular time is known as instantaneous current, i, and it is expressed as,

$$i = \frac{dq}{dt} \tag{1.2}$$

$$dq = i\,dt \tag{1.3}$$

$$q = \int_{t_1}^{t_2} i\,dt \tag{1.4}$$

Equation (1.4) provides the expression for the total charge transferred through a conductor between time t_1 and t_2.

Example 1.1 The expression of charge at a terminal is given by $q = 2t^3 + 5t^2 + t + 1$ C. Find the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(2t^3 + 5t^2 + t + 1) = 6t^2 + 10t + 1 \text{ A} \tag{1.5}$$

At $t = 0.1$ s, the value of the current is,

$$i = 6(0.5)^2 + 10(0.5) + 1 = 7.5 \text{ A} \tag{1.6}$$

Practice Problem 1.1 The total charge entering a terminal is given by $q = 2e^{-2} + 3t^2 + t$ C. Find the current at $t = 0.1$ s.

Example 1.2 A current as shown in Fig. 1.2 passes through a wire. Determine the value of the charge.

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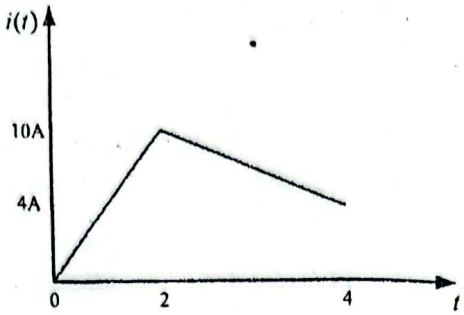


Fig. 1.2 Variation of current with time

Solution:
The expression of current from 0 to 2 s is,

$$i(t) = \frac{10}{2}t = 5t \text{ A} \tag{1.7}$$

The expression of current from 2 to 4 s can be written as,

$$\frac{i(t) - 4}{4 - 10} = \frac{t - 4}{4 - 0} \tag{1.8}$$

$$i(t) - 4 = -\frac{6}{4}(t - 4) \tag{1.9}$$

$$i(t) = 4 - \frac{6}{4}(t - 4) = 10 - 1.5t \tag{1.10}$$

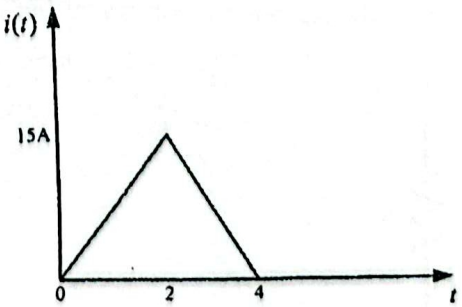
The value of the charge from 0 to 4 s can be calculated as,

$$q = \int_0^2 5t\,dt + \int_2^4 (10 - 1.5t)dt \tag{1.11}$$

$$q = 2.5(2^2 - 0) + 10(4 - 2) - 0.75(4^2 - 2^2) = 21 \text{ C} \tag{1.12}$$

Practice Problem 1.2 The value of an instantaneous current varying with time is shown in Fig. 1.3. Calculate the value of the charge.

Fig. 1.3 Triangular shape of current



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1.4 Direct and Alternating Currents

There are two types of current: direct current, which is abbreviated as DC and alternating current, which is abbreviated as AC [4, 5]. When the magnitude of the current does not change with time, the current is known as direct current, while, in alternating current, the magnitude of the current changes with respect to the time. Both of these currents are shown in Fig. 1.4.

1.5 Conductor, Insulator and Semiconductor

The concept of conductor, insulator and semiconductor can be presented in terms of energy band models from the solid-state physics [6]. The band model consists of two energy bands, valance band and conduction band, and the energy gap between these two bands. The lower energy valance band contains the charges at their lowest energy levels while the highest energy conduction band, being generally empty, contains the energy-excited charges to make conduction. In case of a conductor, a large number of charges populate the conduction band even at room temperature, and as a result, a conductor provides least resistance to the flow of charges. In this case, there is no energy gap in between the valence band and the conduction band, and that means these two energy bands overlap with each other as shown in Fig. 1.5a. For a conductor, the charges in the conduction band are loosely bound with the parent atoms and hence, they can move easily under the influence of external energy source such as electric field.

In the insulator, the energy gap in between the valence and conduction bands is very high as a result, no charge can gain enough energy to jump to the conduction band as shown in Fig. 1.5b. As a result, an insulator provides most resistance to the flow of charges.

A semiconductor falls in between the conductor and insulator and it offers a moderate resistance to flow the charge. A semiconductor has smaller energy gap in between the conduction band and the valence band, when compared to the insulators, as shown in Fig. 1.5c.

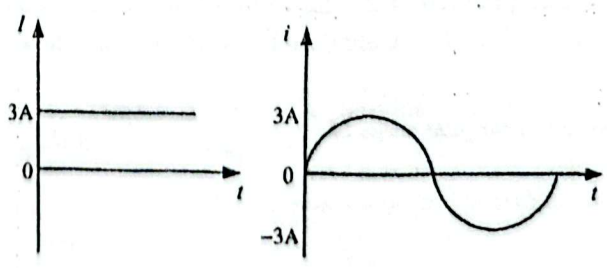


Fig. 1.4 Direct and alternating currents

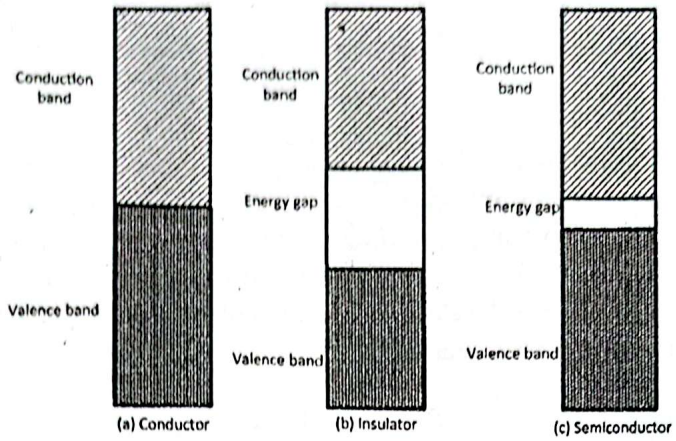


Fig. 1.5 Energy bands conductor, insulator and semiconductor

1.6 Resistance and Conductance

The property of a material, which opposes the flow of electric current through it, is known as resistance. The resistance is represented by the letter R and its SI unit is ohm (Ω). The symbol of a resistor is shown in Fig. 1.6. The resistance of any material is directly proportional to the length (l), and inversely proportional to the cross-sectional area (A) of the material as shown in Fig. 1.7, and can be written as,

$$R \propto l \tag{1.13}$$

$$R \propto \frac{1}{A} \tag{1.14}$$

Combining Eqs. (1.13) and (1.14) yields,

$$R \propto \frac{l}{A} \tag{1.15}$$

$$R = \rho \frac{l}{A} \tag{1.16}$$

where ρ is the proportionality constant which is known as the resistivity of a material and its unit is Ω-m. A good conductor has a very low resistivity, while an insulator has a very high resistivity.

Fig. 1.6 Symbol of the resistor



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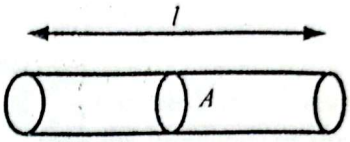


Fig. 1.7 A material with cross-sectional area A

In practice, the common commercial resistors are manufactured in a way so that their resistance values can be determined by the colour bands printed on their surfaces. Different coded values for different colours are shown in Table 1.1. The manufacturing tolerance for each resistor is also colour coded on its surface; Table 1.2 shows these coded values. From the colour bands, the value of a particular resistance can be determined as,

$$R = xy \times 10^z\ \Omega \tag{1.17}$$

where x is the first colour band, closest to the edge of the resistor, y is the second colour band next to x and z is the third colour band. The fourth colour band (if any) represents the tolerance in percent.

Tolerance is the percentage of error in the resistance value that says how much more or less one can expect a resistor's actual measured resistance to be from its stated resistance.

As an example, the resistance value of a resistor shown in Fig. 1.8 can be determined from its colour code, with the help of Tables 1.1 and 1.2 as,

$$R = 52 \times 10^6 = 52\ \text{M}\Omega \tag{1.18}$$

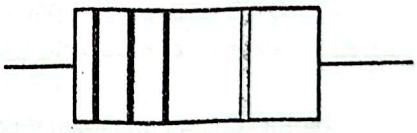


Fig. 1.8 Resistance with different colours

Table 1.1 Resistance colour codes with values

Colours name	Colours	Value
Black		0
Brown		1
Red		2
Orange		3
Yellow		4
Green		5
Blue		6
Violet		7
Grey		8
White		9

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Table 1.2 Tolerance colour codes with values

Colours name	Colours	Value (%)
Orange		0.01
Yellow		0.001
Brown		1
Red		2
Gold		5
Silver		10
No colour		20

The tolerance is calculated as,

$$\text{Tolerance} = 52 \times 10^6 \times 10\% = 5.2\ \text{M}\Omega \tag{1.19}$$

The measured resistance may vary between (52 + 5.2) and (52 − 5.2) MΩ.

Conductance, which is the reciprocal of the resistance, is a characteristic of materials that promotes the flow of electric charge. It is represented by the letter G and its SI unit is Siemens (S). Mathematically, it is expressed as,

$$G = \frac{1}{R} \tag{1.20}$$

Dependence of Resistance on Temperature The resistance of a good conductor such as, copper aluminium, etc., increases with an increase in temperature. Whereas the resistance of electrolyte (*An electrolyte is a substance that produces an electrically conducting solution when dissolved in a polar solvent, such as water*), alloy (*An alloy is a metal, made of the combination of two or more metallic elements*) and insulating material decreases with an increase in temperature. Let us assume that the resistance of a conductor at temperature T and T_0 degree centigrade are R and R_0, respectively. In this case, the change in resistance is,

$$\Delta R = R - R_0 \tag{1.21}$$

While the change in temperature is,

$$\Delta T = T - T_0 \tag{1.22}$$

The ratio of change in resistance to the resistance at T_0 degree centigrade is directly proportional to the change in temperature and it can be expressed as,

$$\frac{R - R_0}{R_0} \propto (T - T_0) \tag{1.23}$$

$$\frac{R - R_0}{R_0} = \alpha(T - T_0) \tag{1.24}$$

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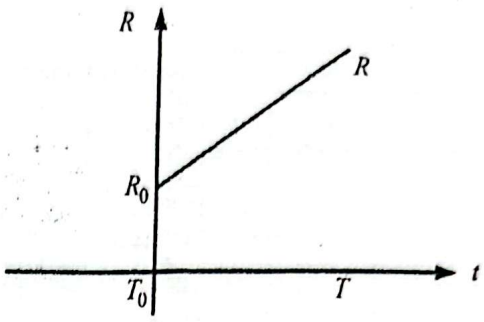


Fig. 1.9 Resistance with different temperatures

$$R - R_0 = \alpha R_0 (T - T_0) \tag{1.25}$$

$$R = [R_0 + \alpha R_0 (T - T_0)] \tag{1.26}$$

$$R = R_0 [1 + \alpha (T - T_0)] \tag{1.27}$$

where α is the proportionality constant which is known as the temperature coefficient of the resistance. Equation (1.27) provides the relationship between the resistance and the temperature, which has been presented in Fig. 1.9. In this figure, the temperature coefficient α represents the slope of the line.

Example 1.3 The length and diameter of a copper wire are found to be 100 and 0.002 m, respectively. Consider that the resistivity of the copper wire is 1.72×10^{-8} Ω-m. Calculate the value of the resistance.

Solution:
The area of the wire is,

$$A = \pi d^2 = \pi (0.002)^2 = 1.26 \times 10^{-5} \text{ m}^2 \tag{1.28}$$

The value of the resistance can be calculated as,

$$R = \rho \frac{l}{A} = 1.72 \times 10^{-8} \times \frac{100}{1.26 \times 10^{-5}} = 0.14\ \Omega \tag{1.29}$$

Practice Problem 1.3 The resistance of a 120 m length aluminium wire is found to be 10 Ω. The resistivity of the aluminium wire is 2.8×10^{-8} Ω-m. Determine the diameter of the wire.

1.7 Voltage

The voltage is one kind of force required to move a charge in the conductor. Voltage is defined as the work done per unit charge when a charge to move it from one point to another point in a conductor. It is represented by the letter *V* and its unit is volts (V) in honour of Italian Scientist Alessandro Giuseppe Antonio Anastasio

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Volta (February 1745–March 1827) who invented electric battery. Mathematically, the voltage is expressed as,

$$v = \frac{dw}{dq} \tag{1.30}$$

In general, the expression of voltage is,

$$V = \frac{W}{Q} \tag{1.31}$$

As common analogy, a water tank can be used to describe the charge, voltage and current. In this analogy, the amount of water in the tank represents the charge, the water pressure represents the voltage and the flow of water through the hose-pipe, which is connected at the bottom of the tank, represents the current as shown in Fig. 1.10.

The voltage difference is the difference in potential between the two points in a conductor, which is also known as potential difference.

Example 1.4 A charge is required to move between the two points in a conductor. In this case, the expression of work done is given by $w = 3q^2 + q + 1$ J. Find the voltage to move the charge $q = 0.5$ C.

Solution:
The expression of voltage is,

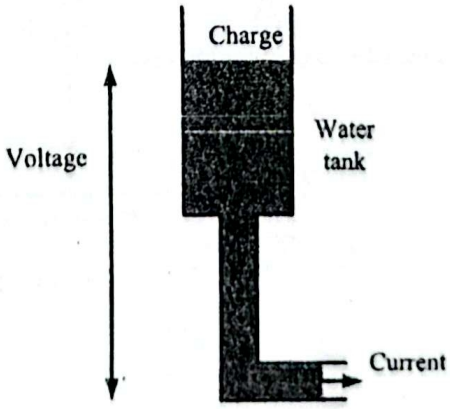
$$v = \frac{dw}{dq} = \frac{d}{dq}(3q^2 + q + 1) = 6q + 1 \text{ V} \tag{1.32}$$

The value of the voltage for $q = 0.5$ C is,

$$v = 6q + 1 = 6 \times 0.5 + 1 = 4 \text{ V} \tag{1.33}$$

Practice Problem 1.4 The work done is given by $w = 5q^2 + 2q$ J when a charge is moved between the two points in a conductor. Determine the value of the charge if the voltage is found to be 10 V.

Fig. 1.10 A tank with water



1.8 Voltage and Current Sources

Any electrical source (voltage or current) provides energy to the elements connected to this source. An ideal voltage source is a two-terminal circuit element that provides a specific magnitude of voltage across its terminals regardless of the current flowing through it. An ideal current source is a two-terminal circuit element that maintains constant current through its terminals regardless of the voltage across those terminals. Ideal voltage and current sources are shown in Fig. 1.11.

An active circuit element usually supplies voltage or current to the passive elements like resistance, inductance and capacitance. Active element is classified as either independent or dependent (controlled) source of energy. An independent voltage or current source is an active element that can generate specified voltage or current without depending on any other circuit elements. The symbols for independent voltage and current sources are shown in Fig. 1.12. A dependent source is an active element whose magnitude is fully dependent on other voltage or current value in the circuit. The symbols for dependent voltage and current sources are shown in Fig. 1.13. The dependent sources are classified into four types, and these are,

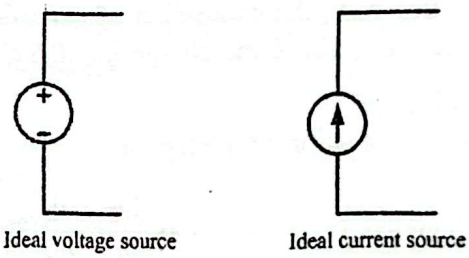


Fig. 1.11 Ideal voltage and current sources

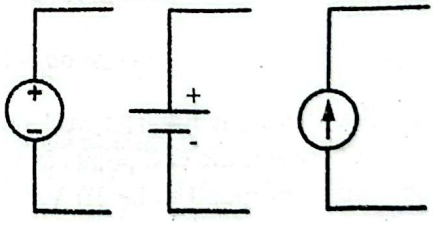


Fig. 1.12 Independent sources

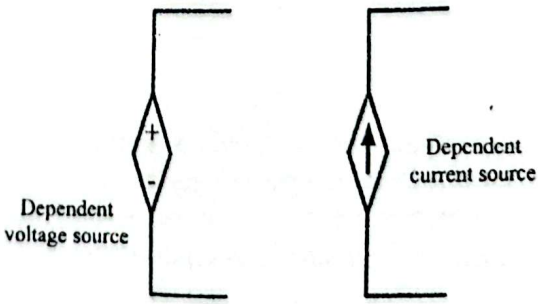


Fig. 1.13 Dependent sources

(1) Voltage-controlled voltage source (VCVS)
(2) Voltage-controlled current source (VCCS)
(3) Current-controlled voltage source (CCVS)
(4) Current-controlled current source (CCCS)

Different controlled (dependent) sources along with the corresponding application circuits are shown in Figs. 1.14, 1.15, 1.16 and 1.17. In Fig. 1.14, the magnitude of the voltage in the VCVS is α times the voltage across any element, and in the corresponding application circuit, this magnitude is equal to 0.2 (the value of α) times the voltage across the resistance R_1.

In the VCCS application circuit in Fig. 1.15, the magnitude of the voltage source depends on the current through resistance R_1. Similar explanation can be offered for CCVS and CCCS and their application circuits are shown in Figs. 1.16 and 1.17, respectively.

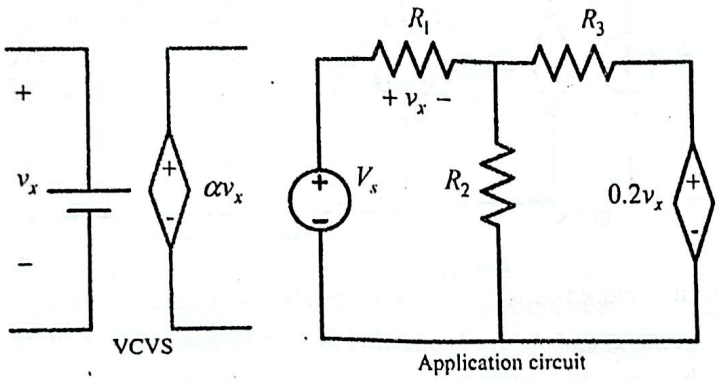


Fig. 1.14 Voltage-controlled voltage source and its application

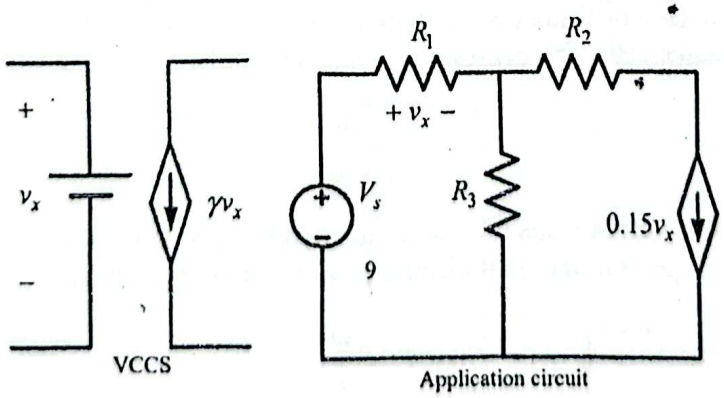


Fig. 1.15 Voltage-controlled current source and its application

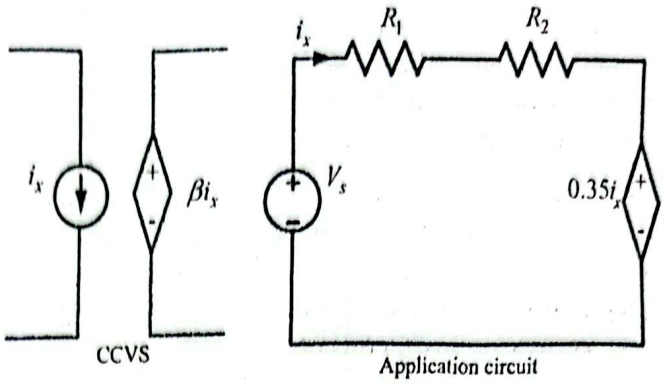


Fig. 1.16 Current-controlled voltage source and its application

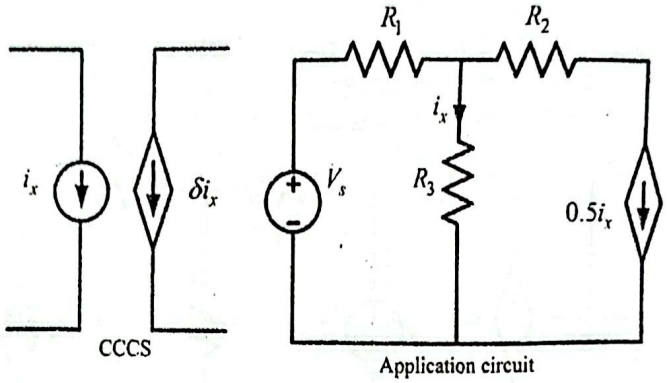


Fig. 1.17 Current-controlled current source and its application

1.9 Electric Power and Energy

The electric power is defined as the rate of receiving or delivering energy from one circuit to another circuit. The power is represented by the letter p and its unit is joules per sec (J/s) or watts (W) in honour to British Scientist James Watt (1736–1819). Mathematically, the expression of power can be written as,

$$p = \frac{dw}{dt} \tag{1.34}$$

where p is the power in watts (W), w is the energy in joules (J) and t is the times in seconds (s). In general, the total electric power can be written as,

$$P = \frac{W}{t} \tag{1.35}$$

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Equation (1.34) can be re-arranged as,

$$p = \frac{dw}{dq}\frac{dq}{dt} \tag{1.36}$$

Substituting Eqs. (1.2) and (1.30) into Eq. (1.36) yields,

$$p = vi \tag{1.37}$$

Equations (1.1) and (1.31) can be re-arranged as,

$$t = \frac{Q}{I} \tag{1.38}$$

$$W = VQ \tag{1.39}$$

Substituting Eqs. (1.38) and (1.39) into Eq. (1.35) yields,

$$P = \frac{W}{t} = \frac{VQ}{\frac{Q}{I}} \tag{1.40}$$

$$P = VI \tag{1.41}$$

Both Eqs. (1.37) and (1.41) show that the power associated to any basic circuit element is the product of the voltage across that element and the current through that element. If the power, associated to a circuit element, has positive sign, then the power is absorbed by that element. Whereas, in a circuit element, the power with negative sign represents that the element delivers power to the other elements. The phenomena of power delivery and power absorption by the any element can be visualized with the aid of Fig. 1.18.

According to the circuit in Fig. 1.18, the expressions of power can be expressed as,

$$P_d = V(-I) = -VI \tag{1.42}$$

$$P_r = VI \tag{1.43}$$

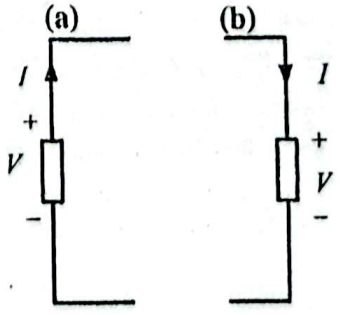


Fig. 1.18 Power delivered and absorbed circuits

The negative sign in Eq. (1.42) indicates that the power is being delivered by the element, while the positive sign in Eq. (1.43) indicates the power is being absorbed by the element.

Energy is the capacity to do work. Since, the SI unit of work is joule (J), and for the same reason, it is represented by the letter w. From Eq. (1.34), the expression of energy can be written as,

$$dw = pdt \tag{1.44}$$

For a time window between t_0 and t, the total energy can be calculated as,

$$w = \int_{t_0}^{t} pdt \tag{1.45}$$

In practice, the energy supplied to the consumers is rated the term of kilowatt hour (kWh). One kWh is defined as the electrical power of 1 kW consumed in an hour, and it is expressed as,

$$E = P \times t \tag{1.46}$$

where E is the electrical energy in kWh, P is the power in kW and t is the time in hour.

Example 1.5 The expression of work done is given by $w = 5t^2 + 4t + 1$ J. Calculate the power at $t = 0.1$ s.

Solution:
The expression of power is,

$$p = \frac{dw}{dt} = \frac{d}{dt}(5t^2 + 4t + 1) = 10t + 4 \text{ W} \tag{1.47}$$

Power at $t = 0.1$ s is,

$$p = 10(0.1) + 4 = 5 \text{ W} \tag{1.48}$$

Example 1.6 A power with an expression of $p = 5 + 5t^2$ W is used from time 0.1 to 0.4 s to complete a specific task. Calculate the corresponding energy.

Solution:

$$\begin{aligned} w &= \int_{0.1}^{t=0.4} (5 + 5t^2)dt = 5[t]_{0.1}^{0.4} + \frac{5}{3}[t^3]_{0.1}^{0.4} \\ &= 5(0.4 - 0.1) + 1.67(0.4^3 - 0.1^3) = 1.61 \text{ J} \end{aligned} \tag{1.49}$$

Example 1.7 A 500 W electric toaster operates for 30 min, a 7 kW DVD player operates for 1 h and a 700 W electric iron operates for 45 min. Find the total energy used, and the associated cost if the energy price is 5 cents per kWh.

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Solution:
The total energy used is,

$$E = \frac{500}{1000} \times \frac{30}{60} + 7 \times 1 + \frac{700}{1000} \times \frac{45}{60} \quad (1.50)$$
$$= 0.500 \times 0.5 + 7 + 0.700 \times 0.75 = 7.78 \text{ kWh}$$

The cost of the energy used is,

$$C = 7.78 \times \frac{5}{100} = 0.39\,\$ \quad (1.51)$$

Practice Problem 1.5 The power consumed at time t, for the work $w = 5t^2 + 4t + 1$ J, is found to be 20 W. Calculate time t.

Practice Problem 1.6 A power of $p = 10e^{-t} + 2$ W is used to complete a task. Calculate the energy for the time interval 0.01 to 0.2 s.

Practice Problem 1.7 Five 10 W energy-saving light bulbs operate for 8 h, a 60 W smart TV operates for 5 h and an 800 W hair dryer operates for 30 min. Calculate the total energy used, and the associated cost if the energy price is 20 cents per kWh.

1.10 Measuring Equipment

Measuring equipment are used to measure the electrical parameters accurately, both analogue and digital metres are available in practice. In an analogue metre, a pointer moves on a scale to read out the reading. Whereas the digital metre can display the reading directly in the form of numerical digits. Some of these widely used metres have been discussed below.

1.10.1 Ohmmeter

An ohmmeter, shown in Fig. 1.19, can measure the resistance of an electrical component. To measure the resistance of any component, it needs to be disconnected from the circuit. Ohmmeter is also used to identify the continuity of a circuit and the correct terminal ends in the application field where long cables are used. In the measurement process, the metre supplies current to the resistance, then measures the voltage drop across that resistance. Finally, the metre gives the value of the resistance.

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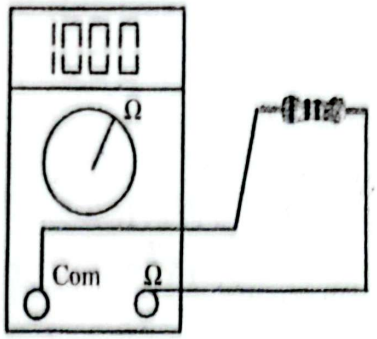


Fig. 1.19 Schematic of an ohmmeter

1.10.2 Ammeter

An ammeter is used to measure the current in the circuit. Ammeter is always connected in series with the circuit under test. This metre offers low internal resistance, which does not affect the actual measurement. The metre symbol and a circuit under test are shown in Fig. 1.20.

1.10.3 Voltmeter

A voltmeter is used to measure the voltage across any circuit element. A voltmeter is always connected in parallel to the circuit element under test. Voltmeters can be designed to measure AC or DC. The internal resistance of a voltmeter is usually kept very high to reduce the current flowing through it to a negligible small amount. Most of the voltmeters have several scales including 0–250, 0–500 V. The metre symbol and a circuit under test are shown in Fig. 1.21.

Fig. 1.20 Symbol and connection diagram of an ammeter

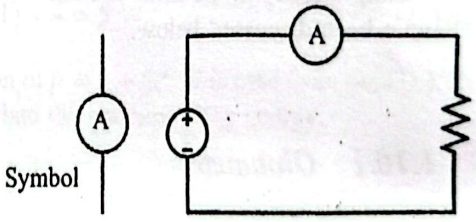
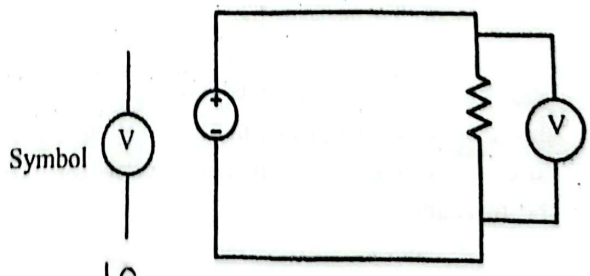


Fig. 1.21 Symbol and connection diagram of a voltmeter



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1.10.4 *Wattmeter*

A wattmeter is used to measure either the power delivered by an electrical source or the power absorbed by a load. As shown in Fig. 1.22, a wattmeter has two coils, the current coil (CC) and the voltage coil (VC).

The current coil with very low resistance is connected in series with the load and responds to the load current, while the voltage coil is connected in parallel to the load circuit and responds to the load voltage. Depending on the test section (source or load), one terminal of each coil is shorted and connected either to the source or to the load as shown in Fig. 1.23. In an AC circuit, wattmeter is widely used to measure the single-phase and three-phase real power as shown in Fig. 1.24.

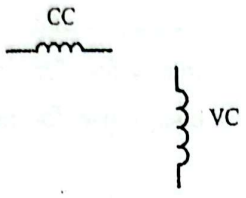


Fig. 1.22 Current coil and voltage coil of a wattmeter

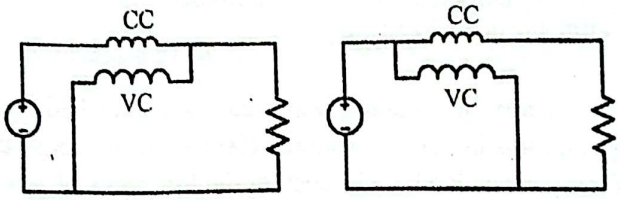


Fig. 1.23 Schematics of a wattmeter connection

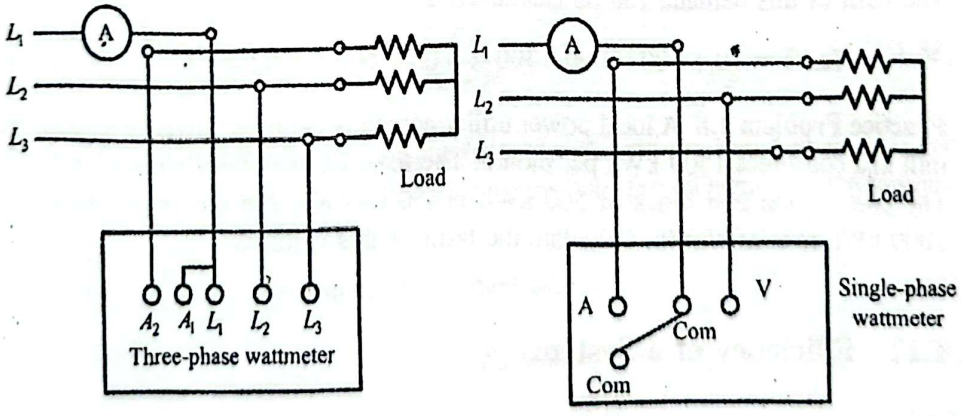


Fig. 1.24 Connection diagrams of three-phase and single-phase wattmeter

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1.11 Electricity Bill

The generated power is usually transmitted and distributed by the power utility company to different types of consumers namely, residential, commercial and industrial for a set price. This price is set by the utility company, by taking into account the energy will charge based on production cost, transmission and distribution cost and the profit margin. The energy consumed by the customers is represented by kWh, based on which the consumers incur the associated costs. The rate at which electrical power is sold to the consumer is generally known as tariff. The main objective of the tariff is to recover the associated costs outlined above. There are different types of tariff namely, two-part tariff, three-part tariff, block rate tariff, maximum demand tariff and power factor tariff. The discussion on these different types of tariff is out of the scope of this text. Generally, the tariff is expressed as,

$$\begin{aligned}\text{Tariff} = a + b \times \text{energy use for first block} \\ + c \times \text{energy use for 2nd block} \\ + d \times \text{energy use for next block}\end{aligned} \tag{1.52}$$

where

a is the fixed charge in \$,
b is the charge (in \$) per kWh for the first block,
c is the charge (in \$) per kWh for the second block,
d is the char kWh for the next block

Example 1.8 The energy of a commercial building is supplied by a local power utility company, and this building consumes 1000 kWh of energy per month. The fixed charge of this demand is \$10. The cost for the first block of 100 kWh is \$ 0.10/kWh, the second block of 300 kWh is \$ 0.05/kWh and the next block is \$ 0.02/kWh. Determine the tariff of this demand.

Solution:
The tariff of this demand can be calculated as,

$$\text{Tariff} = 10 + 100 \times 0.1 + 300 \times 0.05 + 600 \times 0.02 = \$47 \tag{1.53}$$

Practice Problem 1.8 A local power utility company supplies power to a spinning mill that consumes 1500 kWh per month. The fixed charge of this demand is \$100. The cost for the first block of 500 kWh is \$ 0.15/kWh, and the second block of 1000 kWh is \$ 0.02/kWh. Calculate the tariff of this demand.

1.12 Efficiency of a System

The efficiency of a system is defined as the ratio of output power to the input power. The input power is equal to the output power plus the loss as shown in Fig. 1.25. Mathematically, the efficiency η is expressed as,

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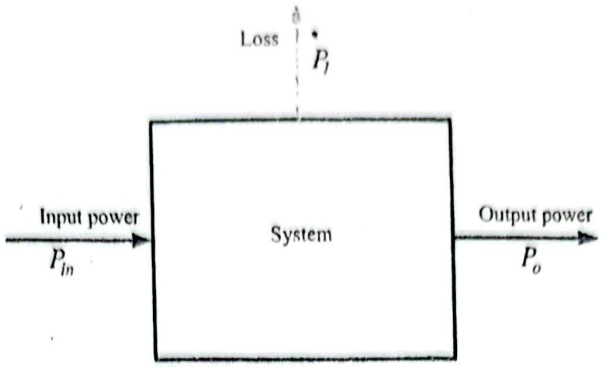


Fig. 1.25 System with different powers

$$\eta = \frac{P_o}{P_{in}} \times 100 \tag{1.54}$$

$$P_{in} = P_o + P_l \tag{1.55}$$

where

P_{in} is the input power in the system in W,
P_o is the output power in the system in W,
P_l is the loss in the system in W.

Substituting Eq. (1.55) into Eq. (1.54) yields,

$$\eta = \frac{P_{in} - P_l}{P_{in}} \times 100 \tag{1.56}$$

$$\eta = \left(1 - \frac{P_l}{P_{in}}\right) \times 100 \tag{1.57}$$

Again, Eq. (1.54) can be written as,

$$\eta = \frac{P_o}{P_o + P_l} \times 100 \tag{1.58}$$

Example 1.9 The input power and the loss of a system are found to be 150 and 30 W, respectively. Determine the output power and the efficiency of the system.

Solution:
The output of the system can be determined as,

$$150 = P_o + 30 \tag{1.59}$$

$$P_o = 150 - 30 = 120 \text{ W} \tag{1.60}$$

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The efficiency is calculated as,

$$\eta = \frac{P_o}{P_{in}} \times 100 = \frac{120}{150} \times 100 = 80\% \tag{1.61}$$

Practice Problem 1.9 The output power and the loss of a system are found to be 100 and 10 W, respectively. Calculate the input power and the efficiency of the system.

1.13 PSpice Fundamentals

In PSpice simulation, user needs to open the schematic of PSpice and select circuit elements like resistance, voltage source, current source and analogue ground as shown in Fig. 1.26. The circuit elements can be rotated by holding the control (ctrl) key, and clicking on the specific element that needs to be rotated. According to circuit configuration, all elements need to be connected using the pencil tool. The value of any circuit element can be changed by double-clicking on it.

Sometimes, the software library does not contain the necessary circuit parts. In this case, the circuit part can be added by clicking on the *Libraries* button and clicking on the *Part Browser Advanced Window* as shown in Fig. 1.27.

For example, to find a DC voltage source, write the part name underneath the *Part Name* option as shown in Fig. 1.28, and browse the *Part Browser Advanced* window. Once the part appears, click on the place and close the button to get this part in the PSpice schematic window. In the PSpice simulation, the ground (*agnd*) of the circuit is very important. The simulation cannot be carried out without the ground connection. All parts of the circuit need to be placed along with the *agnd* and then are connected using the pencil tool as shown in Fig. 1.29. Then, the final circuit under simulation needs to be saved and then is run by clicking on the simulated button to calculate the voltage and current.

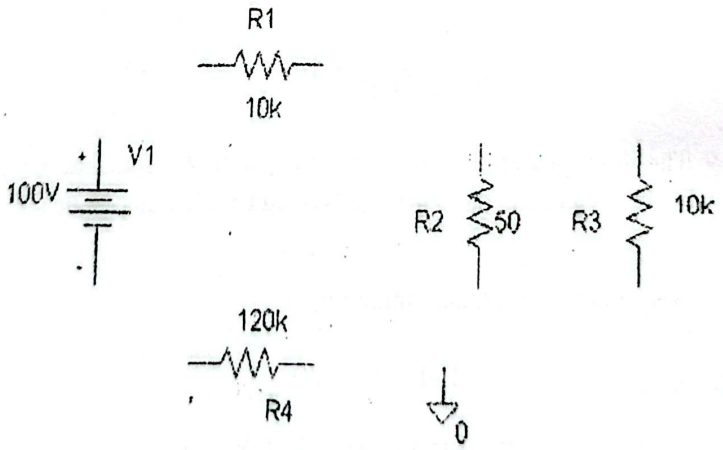


Fig. 1.26 Different circuit elements

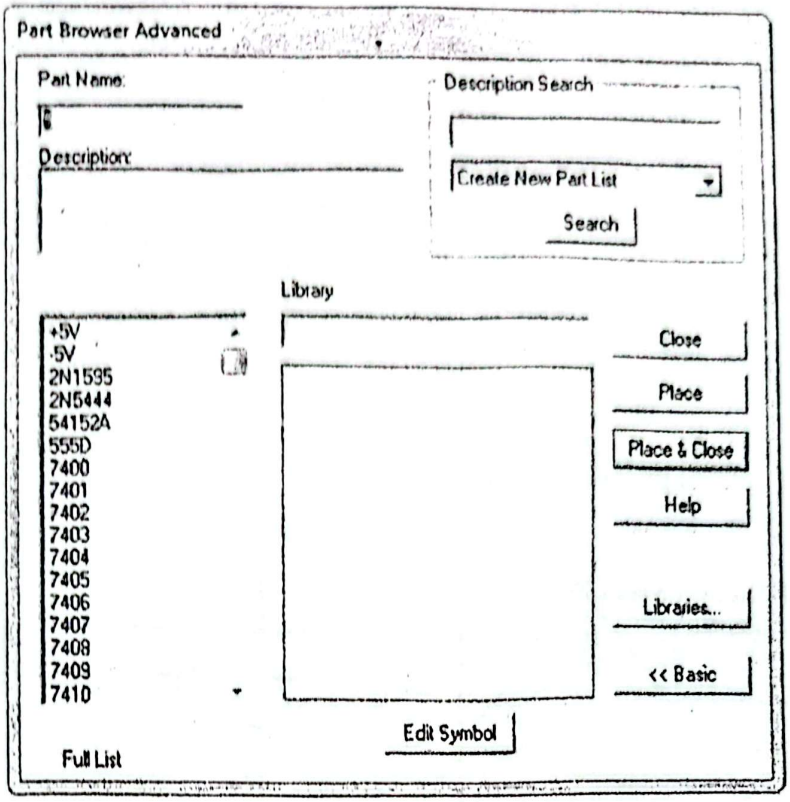
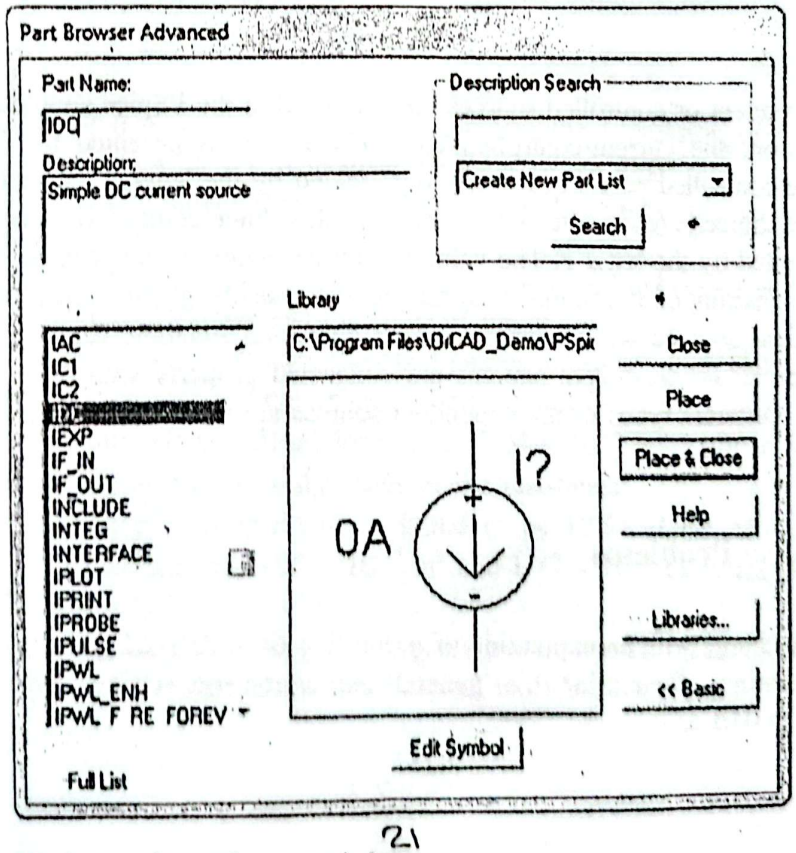


Fig. 1.27 Circuit part browser window



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Fig. 1.28 Missing part in part browser window

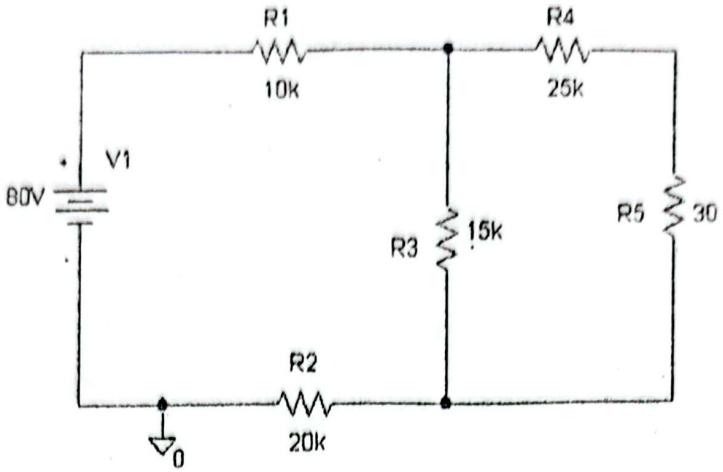
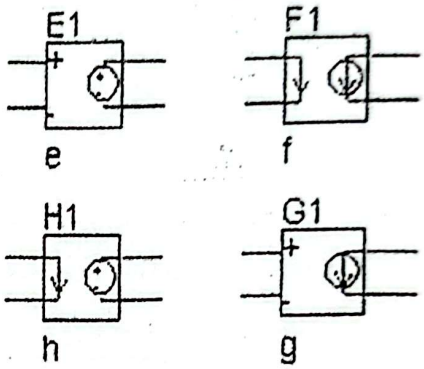


Fig. 1.29 Final for PSpice simulation

Fig. 1.30 Different types of dependent sources



Dependent or controlled sources are also used in the PSpice simulation. In this simulation, the current-controlled current source is represented by the letter *f*, voltage-controlled current source is represented by the letter *g*, current-controlled voltage source is represented by the letter *h* and voltage-controlled voltage source is represented by the letter *e*. The value of any dependent source can be changed by double-clicking on the source. Careful attention must be given while connecting the dependent sources with the other circuit elements. Accurate results can only be obtained if the dependent sources are connected properly with other circuit elements. Different types of the dependent sources are shown in Fig. 1.30.

Exercise Problems

1.1 A charge with an expression of $q = e^{-2t} + 6t^2 + 3t + 2$ C is found in a circuit terminal. Determine the general expression of current and its vale at $t = 0.01$ s.

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Fig. 1.31 Current waveform with times

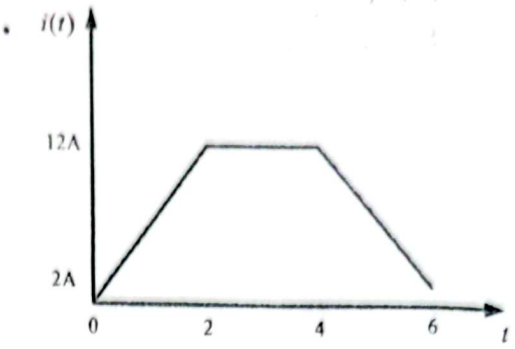
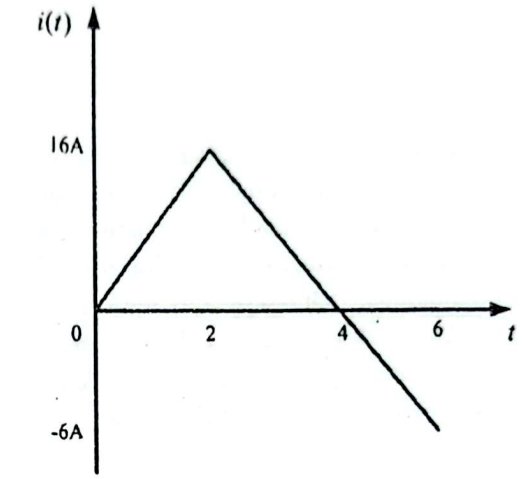


Fig. 1.32 Current waveform with times



1.2 A current of 6 A passes through a wire, where the expression of charge is found to be $q = e^{2t} + 3t$ C. Calculate the value of the time.

1.3 A charge of 8 C moves through a wire for 2 s. Determine the value of the current.

1.4 A current as shown in Fig. 1.31 passes through a wire. Find the expressions of current for different times and the value of the charge.

1.5 Fig. 1.32 shows a current waveform that passes through a wire. Calculate the expression of current and the value of the associated charge.

1.6 The resistivity of a 200 m long and 1.04×10^{-6} m^2 copper wire is 1.72×10^{-8} Ω-m. Calculate the value of the resistance.

1.7 The resistance of a copper wire is found to be 10 Ω. If the area and the resistivity of the wire are 0.95×10^{-7} m^2 and 1.72×10^{-8} Ω-m, respectively, then find the length of the wire.

1.8 The resistivity of a 200 m long aluminium wire is 2.8×10^{-8} Ω-m. Determine the diameter of the wire, if the resistance of the wire is 6 Ω.

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Chapter 2
Electrical Laws

2.1 Introduction

Electrical laws are necessary to analyse any electrical circuit effectively and efficiently by determining different circuit parameters such as current, voltage power and resistance. These laws include Ohms law, Kirchhoff's current and voltage laws, and voltage and current division rules. The knowledge of series and parallel circuit orientations, delta–wye and wye–delta–wye transformations are also required to analyse electrical circuits. In this chapter, different electrical laws, delta–wye and wye–delta–wye transformations, source conversion technique and Wheatstone bridge circuit have been discussed.

2.2 Ohm's Law

Ohm's law is the most important law in the electric circuit analysis that can be applied to any electrical network in any time frame. Ohm's law states that the current flow in a conductor is directly proportional to the voltage across the conductor. A German Physicist, George Simon Ohm (1787–1854), established this relationship between the current and the voltage for a given resistor. According to his name, it is known as Ohm's law. Let a current I flows in a resistor R as shown in Fig. 2.1 resulting in a voltage drop V across the resistor. According to Ohm's law, the relationship between the current and the resultant voltage can be written as,

$$I \propto V \tag{2.1}$$

$$V = IR \tag{2.2}$$

where R is the proportionality constant, which is known as the resistance of the circuit element.

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Fig. 2.1 Current in a resistor

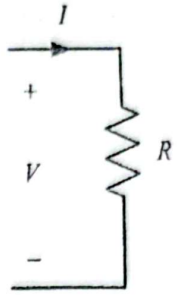
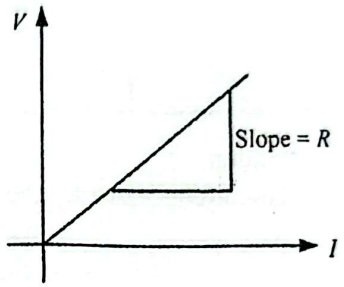


Fig. 2.2 Variation of voltage with current



Equation (2.2) can be expressed as,

$$I = \frac{V}{R} \tag{2.3}$$

From Eq. (2.3), Ohm's law can be restated as follows: the current flowing in a resistor is directly proportional to the voltage across the resistor and inversely proportional to the resistance. According to Ohm's law, if the current increases, the voltage will increase as can be seen in Fig. 2.2, where we assume that the resistance is constant.

Example 2.1 A hairdryer is connected to a 220 V source and draws 3 A current. Determine the value of the resistance.

Solution:
The value of the resistance can be determined as,

$$R = \frac{V}{I} = \frac{220}{3} = 73.33\,\Omega \tag{2.4}$$

Example 2.2 A 10 Ω resistance is connected to a voltage source as shown in Fig. 2.3. Calculate the current drawn and power absorbed by the resistance.

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Fig. 2.3 Circuit for Example 2:2

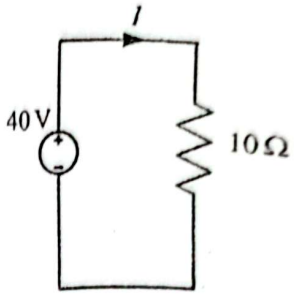
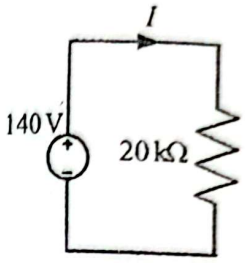


Fig. 2.4 Circuit for Practice Problem 2.2



Solution:
The current drawn by the resistance can be determined as,

$$I = \frac{V}{R} = \frac{40}{10} = 4\,\text{A} \tag{2.5}$$

The power absorbed by the resistance is,

$$P = I^2R = 4^2 \times 10 = 160\,\text{W} \tag{2.6}$$

Practice Problem 2.1
A 600 W electric blender is connected to a 220 V source. Calculate the value of the current and resistance.

Practice Problem 2.2
A 20 kΩ resistance is connected to a voltage source as shown in Fig. 2.4. Find the current.

2.3 Kirchhoff's Current Law

In 1845, German physicist, Gustav Robert Kirchhoff, developed the relationship between different types of currents and voltages in an electrical circuit. These laws, known as Kirchhoff's laws, are used to calculate the current and voltage in an electrical circuit. Kirchhoff's current law states that the algebraic sum of the currents around a node (node refers to any point on a circuit where two or more circuit elements meet) equals to zero. The circuit in Fig. 2.5 illustrates Kirchhoff's current

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Fig. 2.5 Circuit for KCL

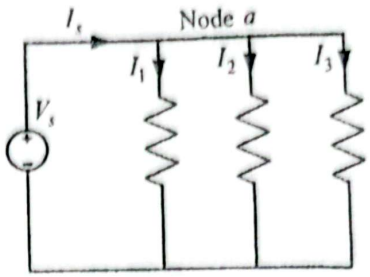
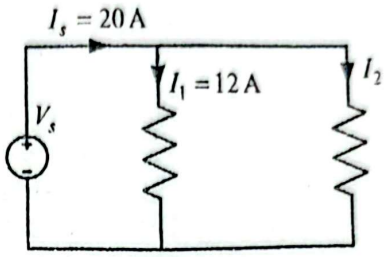


Fig. 2.6 Circuit for Example 2.3



law (KCL). Here, the currents entering node a are considered positive while the currents coming out of the node are considered negative.

From Fig. 2.5, for node a, the following equation can be written:

$$I_s - I_1 - I_2 - I_3 = 0 \tag{2.7}$$

$$I_s = I_1 + I_2 + I_3 \tag{2.8}$$

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}} \tag{2.9}$$

From Eq. (2.9), the KCL can be stated as the sum of the entering currents at a node is equal to the current leaving from the same node.

Example 2.3 The current distribution in a circuit is shown in Fig. 2.6. Determine the value of the unknown current.

Solution:
Applying KCL at the node of the circuit in Fig. 2.6 yields,

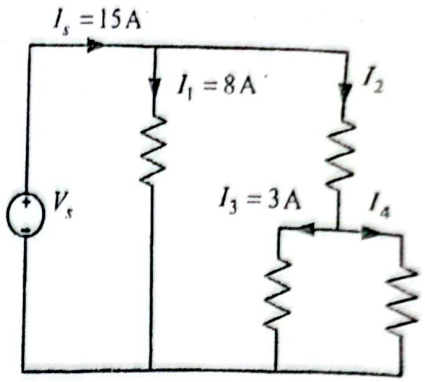
$$20 = 12 + I_2 \tag{2.10}$$

$$I_2 = 20 - 12 = 8\,A \tag{2.11}$$

Practice Problem 2.3
Figure 2.7 shows a circuit with the current distribution. Calculate the values of the unknown currents.

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Fig. 2.7 Circuit for Practice Problem 2.3



2.4 Kirchhoff's Voltage Law

Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltages around any loop (loop refers to a closed path in a circuit) of a circuit is equal to zero. The circuit in Fig. 2.8 is considered to explain the KVL.

Applying KVL to the only loop of the circuit in Fig. 2.8 yields,

$$-V_{s1} - V_{s2} + V_1 + V_2 + V_3 = 0 \tag{2.12}$$

$$V_{s1} + V_{s2} = V_1 + V_2 + V_3 \tag{2.13}$$

$$\sum V_{\text{rises}} = \sum V_{\text{drops}} \tag{2.14}$$

From Eq. (2.14), the KVL can be restated as the sum of the voltage rises is equal to the sum of the voltage drops.

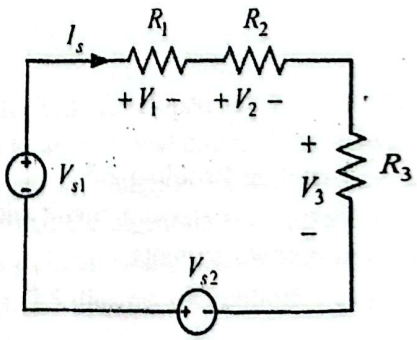
Example 2.4 The voltage distribution of a circuit is shown in Fig. 2.9. Determine the value of V_2, source current I_s and resistance R_3.

Solution:
Applying KVL to the circuit in Fig. 2.9 yields,

$$-40 - 10 + 4 + 6 + V_2 = 0 \tag{2.15}$$

$$V_2 = 40\,V \tag{2.16}$$

Fig. 2.8 Circuit for KVL



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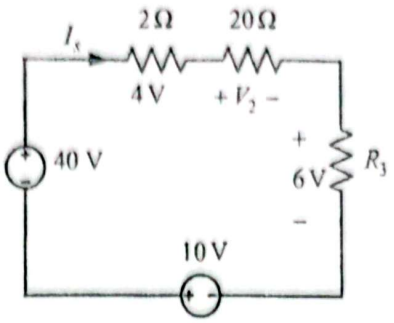


Fig. 2.9 Circuit for Example 2.4

The value of the current in the circuit is determined as,

$$V_2 = I_s \times 4 = 40 \tag{2.17}$$

$$I_s = 10\,\text{A} \tag{2.18}$$

The value of the resistance R_3 is determined as,

$$R_3 = \frac{V_3}{I_s} = \frac{6}{10} = 0.6\,\Omega \tag{2.19}$$

Practice Problem 2.4

Figure 2.10 shows an electrical circuit with different parameters. Calculate the value of the source current, voltage across 6 Ω and 12 Ω resistors.

Example 2.5 An electrical circuit with a voltage-controlled voltage source is shown in Fig. 2.11. Find the power absorbed by the 6 Ω resistor.

Solution:

Applying KVL to the circuit in Fig. 2.11 yields,

$$-40 + 2I_s + 6I_s + 8I_s + 2V_1 = 0 \tag{2.20}$$

$$2I_s - V_1 = 0 \tag{2.21}$$

$$V_1 = 2I_s \tag{2.22}$$

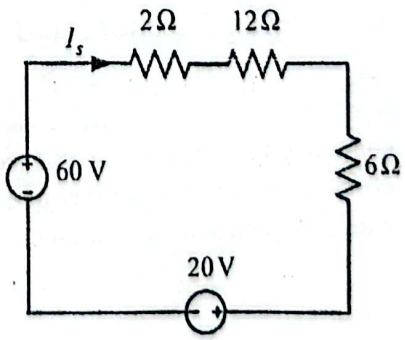


Fig. 2.10 Circuit for Practice Problem 2.4

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Fig. 2.11 Circuit for Example 2.5

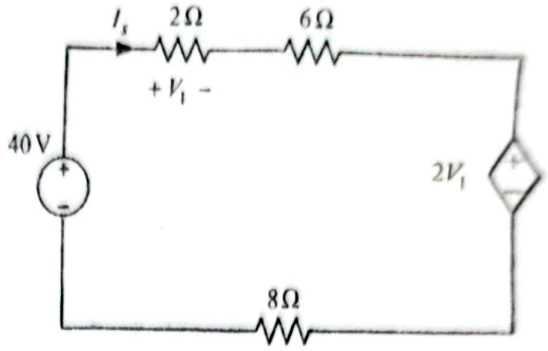
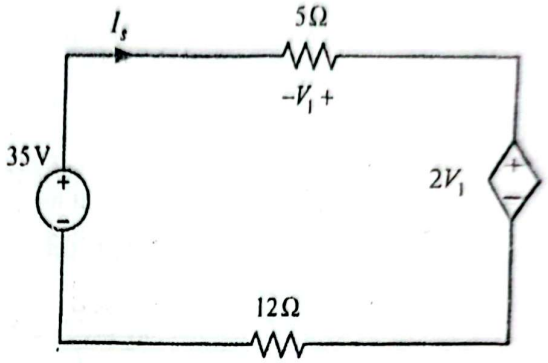


Fig. 2.12 Circuit for Practice Problem 2.5



Substituting Eq. (2.22) into Eq. (2.20) yields,

$$-40 + 2I_s + 6I_s + 8I_s + 2 \times 2I_s = 0 \tag{2.23}$$

$$I_s = \frac{40}{20} = 2\,\text{A} \tag{2.24}$$

The power absorbed by the 6 Ω resistor is,

$$P_{6\Omega} = I_s^2 \times 6 = 2^2 \times 6 = 24\,\text{W} \tag{2.25}$$

Practice Problem 2.5

An electrical circuit with a voltage-controlled voltage source is shown in Fig. 2.12. Calculate the voltage across the 12 Ω resistor.

2.5 Series Resistors and Voltage Division Rule

In a series circuit, the circuit elements such as voltage source, resistors, etc. are connected in an end-to-end connection, where the same current flows through each element. Let us consider that three resistors R_1, R_2, and R_3 are connected in series with a voltage source V_s as shown in the circuit in Fig. 2.13.

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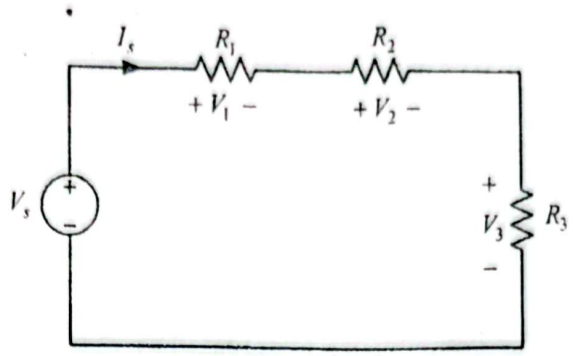


Fig. 2.13 A series circuit with three resistors

If the resulting current I_s flows through this series circuit, according to Ohm's law, as the corresponding voltage drops V_1, V_2, and V_3 across three resistors R_1, R_2, and R_3, respectively, can be written as,

$$V_1 = I_s R_1 \tag{2.26}$$

$$V_2 = I_s R_2 \tag{2.27}$$

$$V_3 = I_s R_3 \tag{2.28}$$

Applying KVL to the circuit in Fig. 2.13 yields,

$$-V_s + V_1 + V_2 + V_3 = 0 \tag{2.29}$$

$$V_s = V_1 + V_2 + V_3 \tag{2.30}$$

Substituting Eqs. (2.26), (2.27) and (2.28) into Eq. (2.30) yields,

$$V_s = I_s R_1 + I_s R_2 + I_s R_3 \tag{2.31}$$

$$V_s = I_s(R_1 + R_2 + R_3) \tag{2.32}$$

$$\frac{V_s}{I_s} = (R_1 + R_2 + R_3) \tag{2.33}$$

$$R_s = (R_1 + R_2 + R_3) \tag{2.34}$$

In Eq. (2.34), R_s represents the series equivalent circuit. Following the similar trend, it can be concluded that if a circuit contains N number of series resistors, then the equivalent resistance can be expressed as,

$$R_s = R_1 + R_2 + R_3 + \cdots + R_N = \sum_{n=1}^{N} R_n \tag{2.35}$$

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The voltage divider rule or voltage division rule is closely related to the series circuit [1, 2]. Substituting Eq. (2.34) to Eq. (2.33) yields the source current in terms of equivalent resistance as,

$$I_s = \frac{V_s}{R_s} \tag{2.36}$$

Equation (2.33) can be rearranged as,

$$I_s = \frac{V_s}{R_1 + R_2 + R_3} \tag{2.37}$$

Substituting Eq. (2.37) into Eqs. (2.26), (2.27) and (2.28) yields,

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V_s \tag{2.38}$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_s \tag{2.39}$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V_s \tag{2.40}$$

By observing Eqs. (2.38) to (2.40), it can be concluded that in a series circuit with resistors and voltage source, the voltage drop across any resistor is equal to the source voltage times the ratio of the resistance of that resistor to the total resistance; this is the voltage divider rule. According to voltage divider rule, the voltage drop V_n across the nth resistor in a circuit with N number of series resistors and voltage source V_s is given by,

$$V_n = \frac{R_n}{R_1 + R_2 + R_3 + \cdots + R_N} V_s \tag{2.41}$$

Example 2.6 A series circuit is shown in Fig. 2.14. Calculate the voltage across each resistor using voltage divider rule.

Solution:
The voltage across 2 Ω resistor is,

$$V_{2\Omega} = \frac{R_1}{R_1 + R_2 + R_3} V_s = \frac{2}{16} \times 80 = 10\,\text{V} \tag{2.42}$$

$$V_{6\Omega} = \frac{R_2}{R_1 + R_2 + R_3} V_s = \frac{6}{16} \times 80 = 30\,\text{V} \tag{2.43}$$

$$V_{8\Omega} = \frac{R_3}{R_1 + R_2 + R_3} V_s = \frac{8}{16} \times 80 = 40\,\text{V} \tag{2.44}$$

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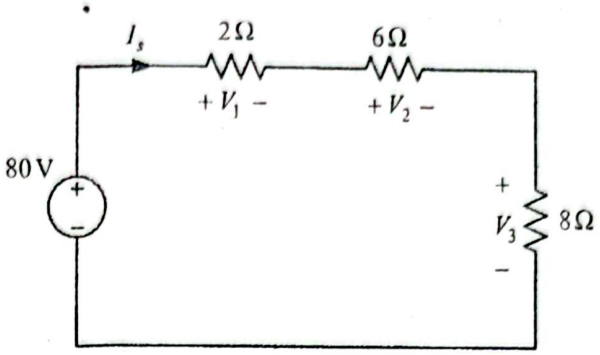


Fig. 2.14 Circuit for Example 2.6

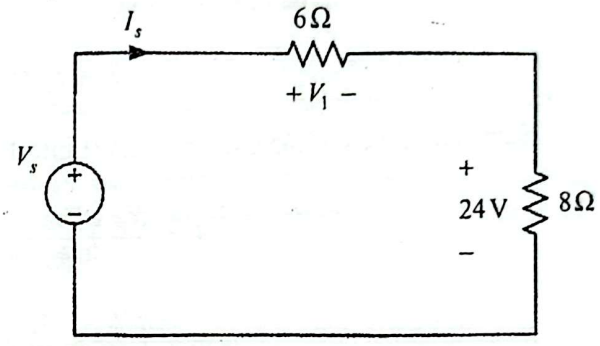


Fig. 2.15 Circuit for Practice Problem 2.6

Practice Problem 2.6

Figure 2.15 shows a series circuit. Use voltage divider rule to find the source voltage and the voltage across the 6 Ω resistor.

2.6 Parallel Resistors and Current Division Rule

Resistors are sometimes connected in parallel in a circuit. Two or more circuit elements are said to be in parallel, when they share same pair of nodes. In a parallel circuit, the voltage across each of the parallel circuit elements is same, but the current through each of the elements might be different. Let us consider three resistors are connected in a parallel to a source voltage V_p as shown in the circuit in Fig. 2.16. In this case, the voltage across each of these resistors will be V_p too.

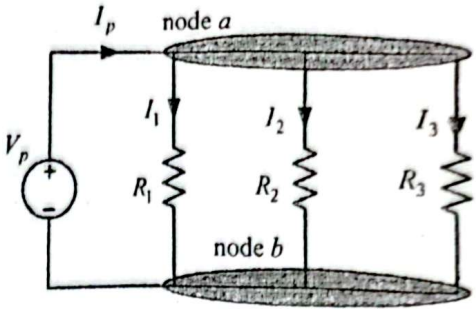
According to Ohm's law, the currents I_1, I_2, and I_2 through the corresponding resistors R_1, R_2, and R_3 can be written as,

$$I_1 = \frac{V_p}{R_1} \tag{2.45}$$

$$I_2 = \frac{V_p}{R_2} \tag{2.46}$$

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Fig. 2.16 Parallel circuit with three resistances



$$I_3 = \frac{V_p}{R_3} \tag{2.47}$$

Applying KCL to the circuit in Fig. 2.16 yields,

$$I_p = I_1 + I_2 + I_3 \tag{2.48}$$

Substituting Eqs. (2.45), (2.46), (2.47) into Eq. (2.48) yields,

$$I_p = \frac{V_p}{R_1} + \frac{V_p}{R_2} + \frac{V_p}{R_3} \tag{2.49}$$

$$I_p = V_p\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \tag{2.50}$$

$$\frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} = \frac{V_p}{I_p} \tag{2.51}$$

$$R_p = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \tag{2.52}$$

Equation (2.52) provides the equivalent resistance for the resistors connected in parallel. For n number of resistors, the above equation can be generalized as:

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \tag{2.53}$$

Let us consider a special case, where two resistors R_1 and R_2 are connected in parallel. In this case, the equivalent resistance R_p will be:

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \tag{2.54}$$

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$$R_p = \frac{1}{\frac{R_1 + R_2}{R_1 R_2}} \tag{2.55}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{2.56}$$

From Eq. (2.56), it can be concluded that the equivalent resistance for two resistors in parallel is equal to the ratio of the product of two resistances to their sum.

If $R_1 = R_2 = R$, then Eq. (2.56) becomes,

$$R_p = \frac{R \times R}{2R} = \frac{R}{2} \tag{2.57}$$

From Eq. (2.57), it is observed that for two resistors, with equal resistance values connected in parallel, will result in the equivalent resistance of half of the value of the single resistor.

Now with the aid of the concept on parallel resistance and the circuit in Fig. 2.17, the current divider rule can be explained. From the circuit in Fig. 2.17, the equivalent resistance R_t can be written as,

$$R_t = \frac{R_1 R_2}{R_1 + R_2} \tag{2.58}$$

The total voltage drop across the two resistors will be,

$$V_t = I_t R_t \tag{2.59}$$

Substituting Eq. (2.58) into Eq. (2.59) yields,

$$V_t = \frac{R_1 R_2}{R_1 + R_2} I_t \tag{2.60}$$

Now that both R_1 and R_2 are in parallel, both the resistors will exhibit the same voltage drop across each of those. In this case, the current in the resistance R_1 will be,

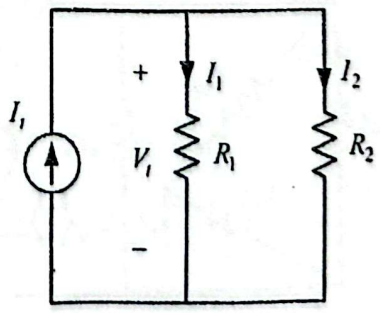


Fig. 2.17 A circuit with two parallel resistors

$$I_1 = \frac{V_t}{R_1} \tag{2.61}$$

Substituting Eq. (2.60) into Eq. (2.61) yields,

$$I_1 = \frac{1}{R_1}\frac{R_1R_2}{R_1+R_2}I_t \tag{2.62}$$

$$I_1 = \frac{R_2}{R_1+R_2}I_t \tag{2.63}$$

The current in the resistance R_2 will be,

$$I_2 = \frac{V_t}{R_2} \tag{2.64}$$

Substituting Eq. (2.60) into Eq. (2.64) yields,

$$I_2 = \frac{1}{R_2}\frac{R_1R_2}{R_1+R_2}I_t \tag{2.65}$$

$$I_2 = \frac{R_1}{R_1+R_2}I_t \tag{2.66}$$

From Eqs. (2.63) and (2.66), it is observed that the current through any resistor in a circuit with two parallel resistors is equal to the total current times the ratio of the opposite resistance to the sum of the two resistances. This is known as current divider rule.

Example 2.7 A series–parallel circuit is shown in Fig. 2.18. Calculate the source current and the branch currents.

Solution:
The total resistance of the circuit is,

$$R_t = 2 + \frac{3 \times (1+5)}{3+6} = 4\,\Omega \tag{2.67}$$

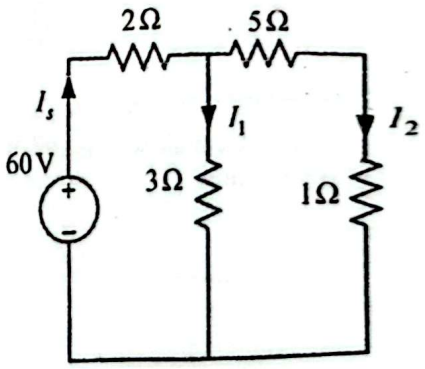


Fig. 2.18 Circuit for Example 2.7

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The value of the source current is,

$$I_s = \frac{60}{4} = 15\,\text{A} \tag{2.68}$$

The branch currents I_1 and I_2 can be calculated as,

$$I_1 = \frac{(5+1)}{6+3} \times 15 = 10\,\text{A} \tag{2.69}$$

$$I_2 = \frac{3}{6+3} \times 15 = 5\,\text{A} \tag{2.70}$$

Example 2.8 Figure 2.19 shows a series–parallel circuit with a jumper wire. Find the total resistance and the source current.

Solution:
In this circuit, 3 Ω and 6 Ω resistors are connected in parallel, and that results in the equivalent resistance as,

$$R_1 = \frac{3 \times 6}{3+6} = 2\,\Omega \tag{2.71}$$

Again, 4 Ω and R_1 (2 Ω) resistors are in series, and this equivalent resistance is parallel to 12 Ω resistor. The overall equivalent resistance is,

$$R_2 = \frac{(4+2) \times 12}{4+2+12} = 4\,\Omega \tag{2.72}$$

Finally, R_2 (4 Ω) and 2 Ω resistors are in series and this equivalent resistance is in parallel to 12 Ω resistor. Finally, this equivalent resistance is in series with 8Ω resistor. The final equivalent circuit resistance is,

$$R_t = 8 + \frac{(4+2) \times 12}{4+2+12} = 12\,\Omega \tag{2.73}$$

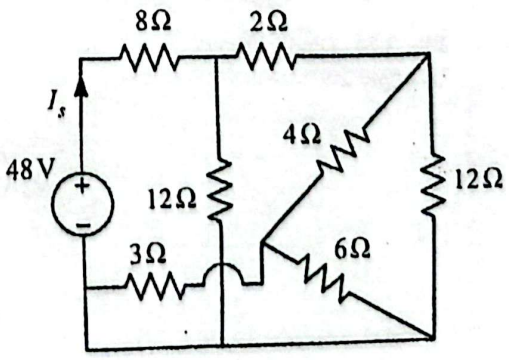


Fig. 2.19 Circuit for Example 2.8

The value of the source current is,

$$I_s = \frac{48}{12} = 4\,\text{A} \tag{2.74}$$

Example 2.9 A circuit with a current-controlled current source is shown in Fig. 2.20. Determine the branch currents and power absorbed by the 4 Ω resistor.

Solution:
Applying KCL to the circuit in Fig. 2.20 yields,

$$I_x + I_1 + 2I_x = 14 \tag{2.75}$$

Applying Ohm's law to the circuit in Fig. 2.20 yields,

$$I_x = \frac{V_s}{2} \tag{2.76}$$

$$I_1 = \frac{V_s}{4} \tag{2.77}$$

Substituting Eqs. (2.76) and (2.77) into Eq. (2.75) yields,

$$14 = 3\frac{V_s}{2} + \frac{V_s}{4} \tag{2.78}$$

$$14 = \frac{7V_s}{4} \tag{2.79}$$

$$V_s = \frac{14 \times 4}{7} = 8\,\text{V} \tag{2.80}$$

Substituting Eq. (2.80) into Eqs. (2.76) and (2.77) yields,

$$I_x = \frac{8}{2} = 4\,\text{A} \tag{2.81}$$

$$I_1 = \frac{8}{4} = 2\,\text{A} \tag{2.82}$$

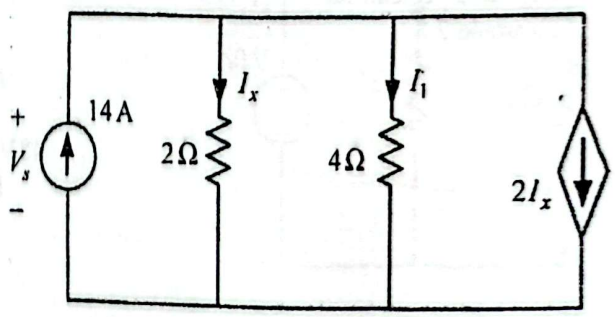


Fig. 2.20 Circuit for Example 2.9

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The power absorbed by the 4 Ω resistance is,

$$P_{4\Omega} = 2^2 \times 4 = 16\,\text{W} \tag{2.83}$$

Practice Problem 2.7

A series–parallel circuit is shown in Fig. 2.21. Calculate the equivalent resistance and the current through 8 Ω resistor.

Practice Problem 2.8

A series–parallel circuit with two jumper wires is shown in Fig. 2.22. Determine the equivalent circuit resistance and the source current.

Practice Problem 2.9

A circuit with a voltage-controlled current source is shown in Fig. 2.23. Calculate the current I_1 and the power absorbed by the 3 Ω resistor.

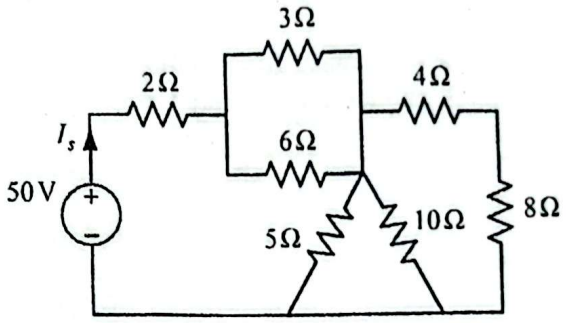


Fig. 2.21 Circuit for Practice Problem 2.7

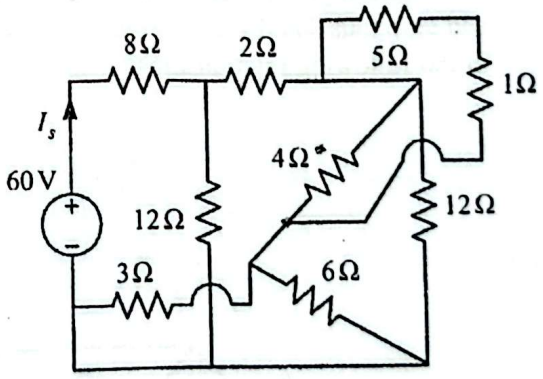


Fig. 2.22 Circuit for Practice Problem 2.8

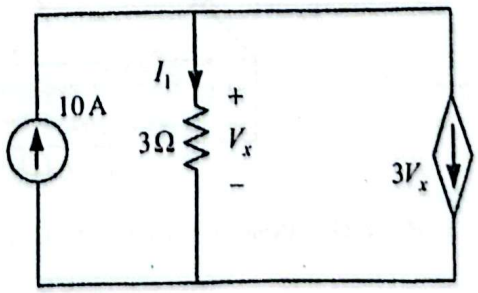


Fig. 2.23 Circuit for Practice Problem 2.9

2.7 Delta–Wye Transformation

Resistors are sometimes either not connected in series or in parallel. In this case, a special technique needs to be used for simplifying an electrical circuit. This technique is known as delta–wye or wye–delta transformation. Consider three resistors which are connected in delta (Δ) and wye (Y) forms as shown in Fig. 2.24.

For delta connection, the total resistances between terminals 1 and 2, terminals 2 and 3, and terminals 3 and 1 are,

$$R_{12(\Delta)} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.84}$$

$$R_{23(\Delta)} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.85}$$

$$R_{31(\Delta)} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.86}$$

For wye connection, the total resistances between terminals 1 and 2, terminals 2 and 3, and terminals 3 and 1 are,

$$R_{12(Y)} = R_1 + R_2 \tag{2.87}$$

$$R_{23(Y)} = R_2 + R_3 \tag{2.88}$$

$$R_{31(Y)} = R_3 + R_1 \tag{2.89}$$

Electrically, the total resistances between terminals 1 and 2, terminals 2 and 3, and terminals 3 and 1 are the same for delta and wye connections [3–5]. This results in the following relationships:

$$R_{12(Y)} = R_{12(\Delta)} \tag{2.90}$$

$$R_{23(Y)} = R_{23(\Delta)} \tag{2.91}$$

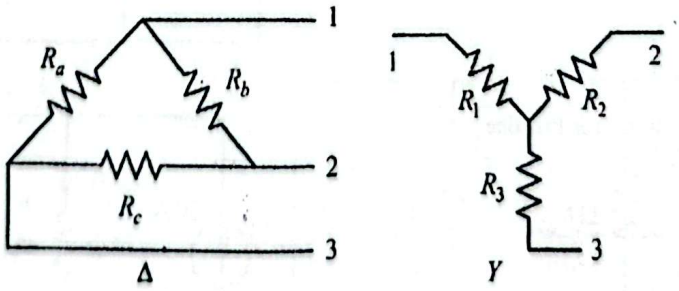


Fig. 2.24 Circuit for delta–wye transformation

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$$R_{31(Y)} = R_{31(\Delta)} \tag{2.92}$$

Substituting Eqs. (2.84) and (2.87) into Eq. (2.90) yields,

$$R_1 + R_2 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.93}$$

Substituting Eqs. (2.85) and (2.88) into Eq. (2.91) yields,

$$R_2 + R_3 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.94}$$

And, substituting Eqs. (2.86) and (2.89) into Eq. (2.92) yields,

$$R_3 + R_1 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.95}$$

Adding Eqs. (2.93), (2.94) and (2.95) yields,

$$2(R_1 + R_2 + R_3) = \frac{2(R_aR_b + R_bR_c + R_cR_a)}{R_a + R_b + R_c} \tag{2.96}$$

$$R_1 + R_2 + R_3 = \frac{R_aR_b + R_bR_c + R_cR_a}{R_a + R_b + R_c} \tag{2.97}$$

Subtracting Eqs. (2.94), (2.95) and (2.93) from Eq. (2.97) individually yields,

$$R_1 = \frac{R_aR_b}{R_a + R_b + R_c} \tag{2.98}$$

$$R_2 = \frac{R_bR_c}{R_a + R_b + R_c} \tag{2.99}$$

$$R_3 = \frac{R_cR_a}{R_a + R_b + R_c} \tag{2.100}$$

2.8 Wye–Delta Transformation

To transform wye-connected resistors into delta-connected ones; the formulae can be derived from Eqs. (2.98), (2.99) and (2.100). Multiplying Eq. (2.98) by (2.99), Eq. (2.99) by (2.100) and Eq. (2.100) by (2.98) yields,

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$$R_1R_2 = \frac{R_aR_b^2R_c}{(R_a+R_b+R_c)^2} \tag{2.101}$$

$$R_2R_3 = \frac{R_aR_bR_c^2}{(R_a+R_b+R_c)^2} \tag{2.102}$$

$$R_3R_1 = \frac{R_a^2R_bR_c}{(R_a+R_b+R_c)^2} \tag{2.103}$$

Adding Eqs. (2.101), (2.102) and (2.103) yields,

$$R_1R_2+R_2R_3+R_3R_1 = \frac{R_aR_b^2R_c+R_aR_bR_c^2+R_a^2R_bR_c}{(R_a+R_b+R_c)^2} \tag{2.104}$$

$$R_1R_2+R_2R_3+R_3R_1 = \frac{R_aR_bR_c(R_a+R_b+R_c)}{(R_a+R_b+R_c)^2} \tag{2.105}$$

$$R_1R_2+R_2R_3+R_3R_1 = \frac{R_aR_bR_c}{R_a+R_b+R_c} \tag{2.106}$$

Dividing Eq. (2.106) by Eqs. (2.99), (2.100) and (2.98) individually yields,

$$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} \tag{2.107}$$

$$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} \tag{2.108}$$

$$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} \tag{2.109}$$

Example 2.10 A delta–wye circuit is shown in Fig. 2.25. Calculate the total circuit resistance, the source current and the power absorbed by the 4 Ω resistor.

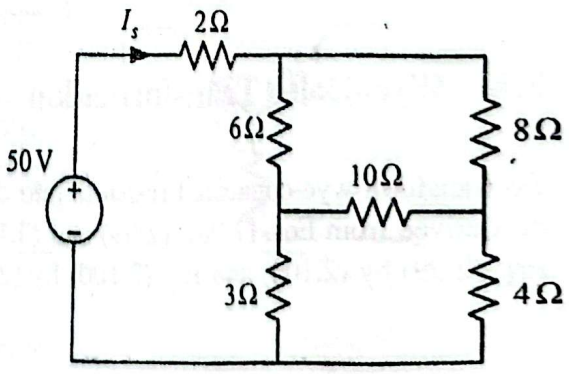


Fig. 2.25 Circuit for Example 2.10

Solution:
Converting the upper part delta circuit to wye circuit as shown in Fig. 2.26. The values of the resistors are,

$$R_1 = \frac{6 \times 8}{6+10+8} = 2\,\Omega \tag{2.110}$$

$$R_2 = \frac{10 \times 8}{6+10+8} = 3.33\,\Omega \tag{2.111}$$

$$R_3 = \frac{10 \times 6}{6+10+8} = 2.5\,\Omega \tag{2.112}$$

Now, the circuit in Fig. 2.26 can be redrawn as shown in Fig. 2.27. The R_3 and $3\,\Omega$ are in series, and R_2 and $4\,\Omega$ are in series, then in parallel to each other. In this case, the equivalent resistance is,

$$R_4 = \frac{(R_2+4)(R_3+3)}{R_2+4+R_3+3} = \frac{(3.33+4)(2.5+3)}{3.33+4+2.5+3} = 3.14\,\Omega \tag{2.113}$$

From Fig. 2.27, the total circuit resistance is determined as,

$$R_t = 2+R_1+R_4 = 2+2+3.14 = 7.14\,\Omega \tag{2.114}$$

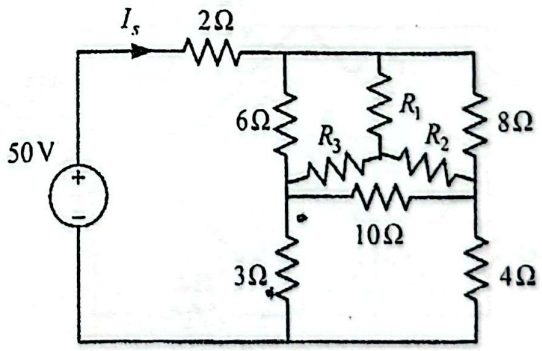


Fig. 2.26 Circuit for conversion delta to wye

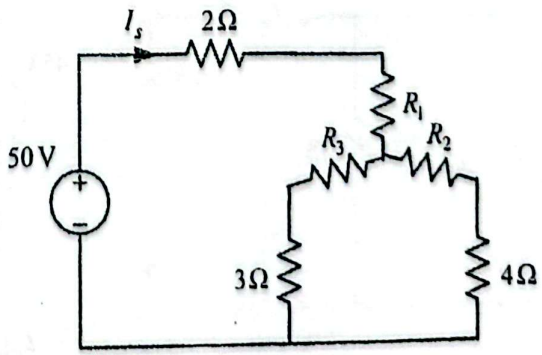


Fig. 2.27 A series–parallel circuit for Example 2.10

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The value of the source current is,

$$I_s = \frac{50}{7.14} = 7\,\text{A} \tag{2.115}$$

The current through 4 Ω resistor is,

$$I_{4\Omega} = 7 \times \frac{(3+2.5)}{5.5+3.33+4} = 3\,\text{A} \tag{2.116}$$

The power absorbed by 4 Ω resistor is,

$$P_{4\Omega} = I_{4\Omega}^2 \times 4 = 3^2 \times 4 = 36\,\text{W} \tag{2.117}$$

Example 2.11 Figure 2.28 shows a delta–wye circuit. Determine the total circuit resistance and the source current.

Solution:
Convert the wye connection to a delta connection as shown in Fig. 2.29. The values of the resistances are,

$$R_1 = \frac{7 \times 10 + 10 \times 3 + 3 \times 7}{3} = 40.33\,\Omega \tag{2.118}$$

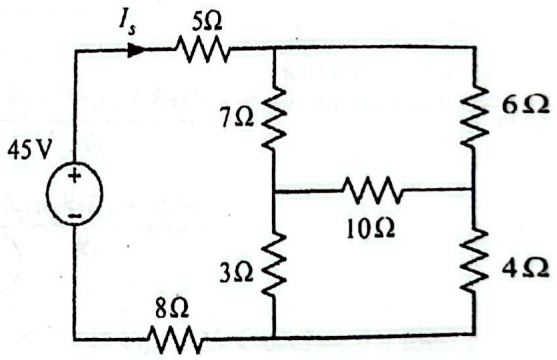


Fig. 2.28 Circuit for Example 2.11

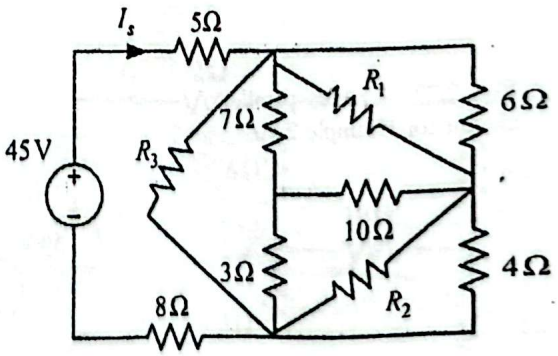


Fig. 2.29 Circuit conversion from wye to delta

$$R_2 = \frac{7 \times 10 + 10 \times 3 + 3 \times 7}{7} = 17.29\,\Omega \tag{2.119}$$

$$R_3 = \frac{7 \times 10 + 10 \times 3 + 3 \times 7}{10} = 12.1\,\Omega \tag{2.120}$$

From Fig. 2.30, it is seen that R_1 and $6\,\Omega$ are in parallel, and R_2 and $4\,\Omega$ are in parallel, and then they are in series. The value of the equivalent resistance R_4 is,

$$R_4 = \frac{R_1 \times 6}{R_1 + 6} + \frac{R_2 \times 4}{R_2 + 4} = \frac{40.33 \times 6}{40.33 + 6} + \frac{17.29 \times 4}{17.29 + 4} = 8.47\,\Omega \tag{2.121}$$

The R_3 and R_4 ($8.47\,\Omega$) resistors are in parallel, then in series with $5\,\Omega$ and $8\,\Omega$ resistors as shown in Fig. 2.31. Finally, the equivalent circuit resistance is,

$$R_t = \frac{12.1 \times 8.47}{12.1 + 8.47} + 5 + 8 = 17.98\,\Omega \tag{2.122}$$

The value of the source current can be determined as,

$$I_s = \frac{45}{17.98} = 2.5\,\text{A} \tag{2.123}$$

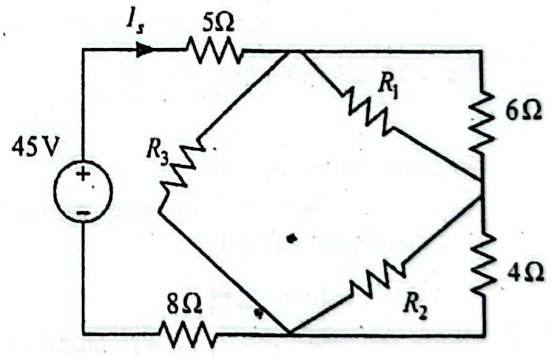


Fig. 2.30 First series–parallel circuit for Example 2.11

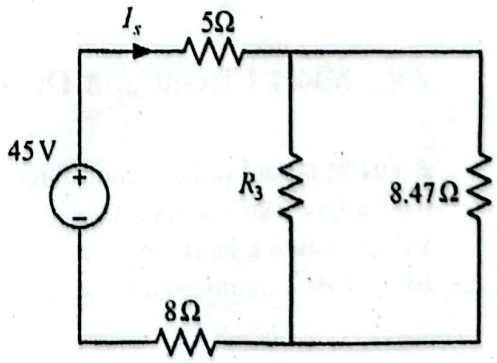


Fig. 2.31 Second series–parallel circuit for Example 2.11

45

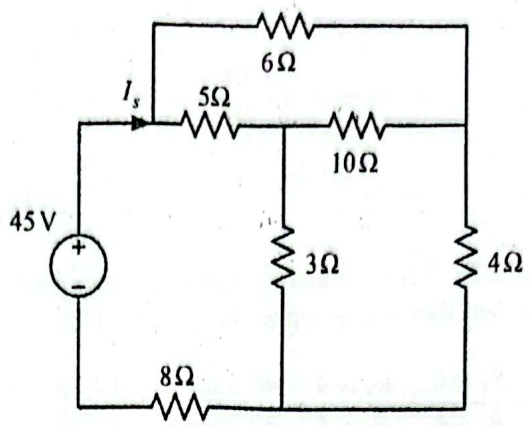


Fig. 2.32 Circuit for Practice Problem 2.10

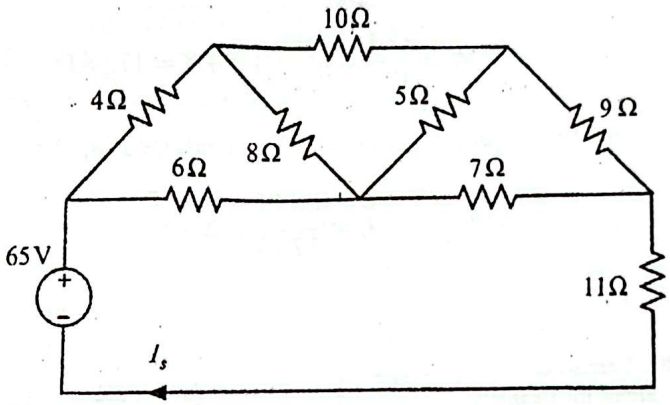


Fig. 2.33 Circuit for Example 2.11

Practice Problem 2.10
Figure 2.32 shows a delta–wye circuit. Calculate the equivalent circuit resistance and the source current.

Practice Problem 2.11
Figure 2.33 shows a delta–wye circuit. Determine the equivalent circuit resistance and the source current.

2.9 Short Circuit and Open Circuit

A circuit is said to be a short circuit when this allows current to flow through an inadvertent path between two nodes with a negligible low (zero) resistance. The voltage across a short circuit is zero. The short-circuit condition can happen both in DC and AC circuits. When two terminals of a dry cell or an automobile battery are

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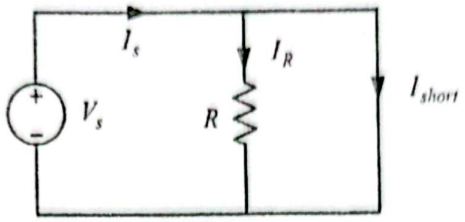


Fig. 2.34 A resistance with a short circuit

shorted by a piece of conducting wire, then the battery will discharge through the wire, which in turn heats up the wire. The principle of an arc welding is based on the application of heating due to a short circuit. Figure 2.34 shows a circuit to explain the short-circuit phenomena. Applying current divider rule the currents I_R and I_{short} are found to be,

$$I_R = I_s \times \frac{0}{R+0} = 0 \tag{2.124}$$

$$I_{short} = I_s \times \frac{R}{R+0} = I_s \tag{2.125}$$

The voltage in the short circuit is,

$$V_{short} = I_{short} \times R_{short} = I_s \times 0 = 0 \tag{2.126}$$

where R_{short} represents the resistance of the short-circuit branch.

A circuit is said to be an open circuit when it contains an infinite resistance between two nodes. Figure 2.35 illustrates the concept of an open circuit. From Fig. 2.34, the expression of the current can be written as,

$$I_s = \frac{V_s}{R} = \frac{V_s}{\infty} = 0 \tag{2.127}$$

Example 2.12 A short and open circuit is shown in Fig. 2.36. Determine the equivalent circuit resistance, and the source current when the terminals *a* and *b* are short and open circuited.

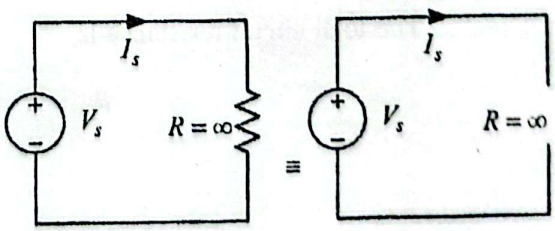
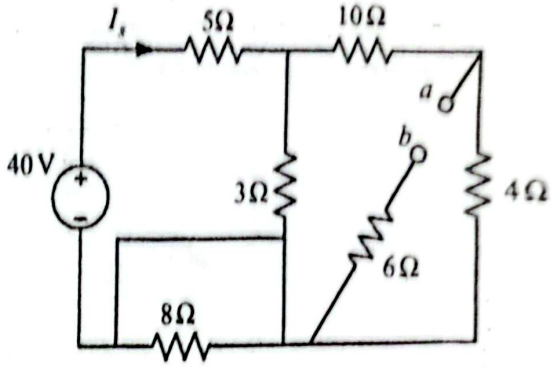


Fig. 2.35 An open

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Fig. 2.36 Circuit for Example 2.12



Solution:

Consider that the terminals *a* and *b* are short circuited. Then, the resistances 6 Ω and 4 Ω are in parallel, and then in series with 10 Ω resistance. The value of the resistance is,

$$R_1 = \frac{4 \times 6}{4+6} + 10 = 12.4\,\Omega \tag{2.128}$$

Again, 12.4 Ω and 3 Ω are in parallel. In this case, the value of the resistance is,

$$R_2 = \frac{12.4 \times 3}{12.4+3} = 2.42\,\Omega \tag{2.129}$$

The total circuit resistance is,

$$R_{ts} = 5 + 2.42 = 7.42\,\Omega \tag{2.130}$$

The value of the source current is,

$$I_{ss} = \frac{40}{7.42} = 5.39\,\text{A} \tag{2.131}$$

Again, consider the terminals *a* and *b* are open circuited. The resistances 4 Ω and 10 Ω are in series, then in parallel with 3 Ω resistance. The values of the resistance are,

$$R_3 = 4 + 10 = 14\,\Omega \tag{2.132}$$

$$R_4 = \frac{14 \times 3}{14+3} = 2.47\,\Omega \tag{2.133}$$

The total circuit resistance is,

$$R_{to} = 5 + 2.47 = 7.47\,\Omega \tag{2.134}$$

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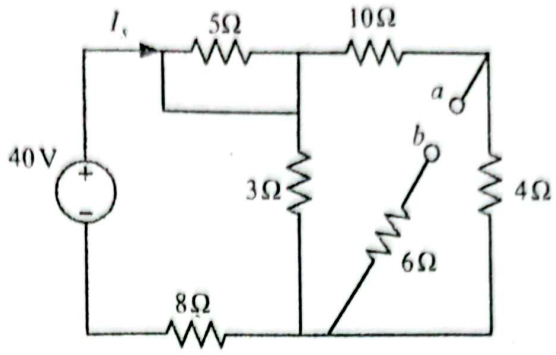


Fig. 2.37 Circuit for Practice Problem 2.12

The value of the source current is,

$$I_{so} = \frac{40}{7.47} = 5.35\,\text{A} \tag{2.135}$$

Practice Problem 2.12
Figure 2.37 shows a short and open circuit. Find the equivalent circuit resistance and the source current.

2.10 Source Configuration

The voltage and current sources are usually connected either in series or in parallel configurations. There are six cells in an automobile battery where each cell provides a voltage of two volts.

These cells are connected in series to generate a voltage of twelve volts. Two voltage sources are connected in series as shown in Fig. 2.38. Consider the voltage source V_{s1} is greater than the voltage source V_{s2}. Then, the following equations from Fig. 2.38 can be written:

$$V_a = V_{s1} + V_{s2} \tag{2.136}$$

$$V_b = V_{s1} - V_{s2} \tag{2.137}$$

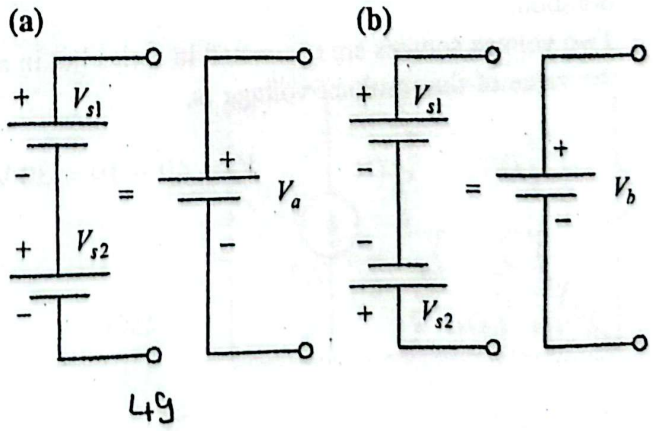


Fig. 2.38 Voltage sources are in series

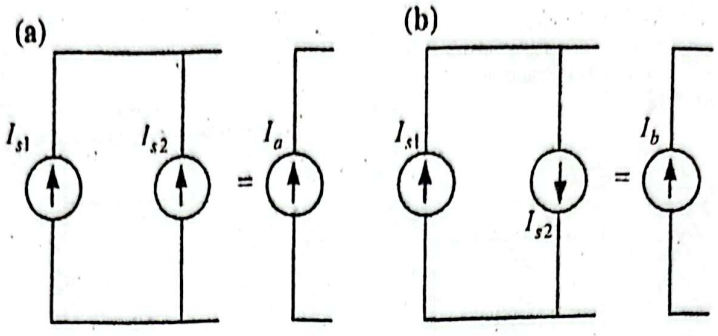
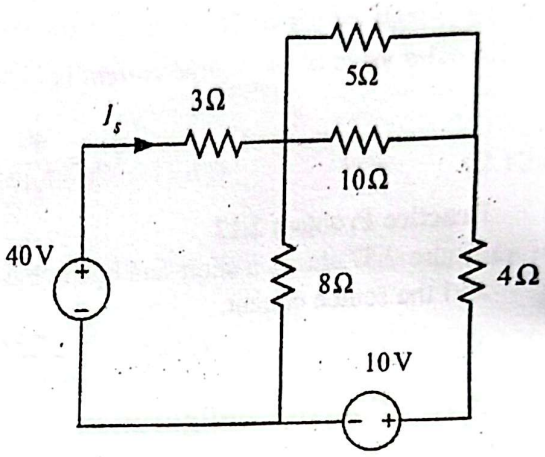


Fig. 2.39 Current sources are in parallel

Fig. 2.40 Circuit for Example 2.13



The current sources are not recommended for series connection, but they are usually connected in a parallel in the circuit as shown in Fig. 2.39.

Consider the current source I_{s1} is greater than the current source I_{s2}. From Fig. 2.39, the following equations can be written:

$$I_a = I_{s1} + I_{s2} \tag{2.138}$$

$$I_b = I_{s1} - I_{s2} \tag{2.139}$$

Example 2.13 A series–parallel circuit with two voltage sources is shown in Fig. 2.40. Calculate the equivalent circuit resistance, source current and the power absorbed by 10 Ω resistor.

Solution:

Two voltage sources are connected in series but in an opposite direction. Therefore, the value of the resultant voltage is,

$$V_s = 40 - 10 = 30\,\text{V} \tag{2.140}$$

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The 5 Ω and 10 Ω resistors are in parallel, and then in series with 4 Ω. The value of the resistance is,

$$R_1 = \frac{5 \times 10}{5+10} + 4 = 7.33\,\Omega \tag{2.141}$$

R_1 (7.33 Ω) from Eq. (2.141) is in parallel with 8 Ω resistor, and then in series with 3 Ω resistance. The equivalent circuit resistance is,

$$R_t = \frac{7.33 \times 8}{7.33+8} + 3 = 6.83\,\Omega \tag{2.142}$$

The value of the source current is,

$$I_s = \frac{30}{6.83} = 4.39\,\Omega \tag{2.143}$$

The current in the 4 Ω resistor is,

$$I_{4\Omega} = 4.39 \times \frac{8}{8+4+3.33} = 2.29\,\text{A} \tag{2.144}$$

The current through 10 Ω resistor is,

$$I_{10\Omega} = 2.29 \times \frac{5}{5+10} = 0.76\,\text{A} \tag{2.145}$$

Power absorbed by 10 Ω resistor is,

$$P_{10\Omega} = 0.76^2 \times 10 = 5.78\,\text{W} \tag{2.146}$$

Example 2.14 A parallel circuit with two current sources is shown in Fig. 2.41. Find the current in the 8 Ω resistor.

Solution:
The value of the equivalent current source is,

$$I_s = 20 - 8 = 12\,\text{A} \tag{2.147}$$

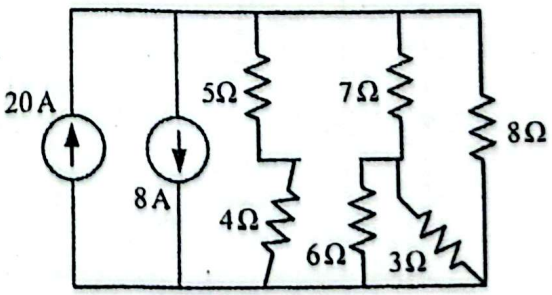


Fig. 2.41 Circuit for Example 2.14

The 6 Ω and 3 Ω resistors are in parallel, then series with 7 Ω resistor. Here, the equivalent resistance is,

$$R_1 = \frac{3 \times 6}{3+6} + 7 = 9\,\Omega \tag{2.148}$$

Then, R_1 (9 Ω) is in parallel with $5+4 = 9\,\Omega$ resistor, and the value of the equivalent resistance is,

$$R_2 = \frac{9 \times 9}{9+9} = 4.5\,\Omega \tag{2.149}$$

The current through 8 Ω resistor is,

$$I_{8\Omega} = 12 \times \frac{4.5}{4.5+8} = 4.32\,\text{A} \tag{2.150}$$

Practice Problem 2.13

Figure 2.42 shows a series–parallel circuit with two voltage sources. Determine the equivalent resistance, source current and the current in the 6 Ω resistor.

Practice Problem 2.14

Figure 2.43 shows a parallel circuit with two current sources. Determine the current in the 8 Ω resistance.

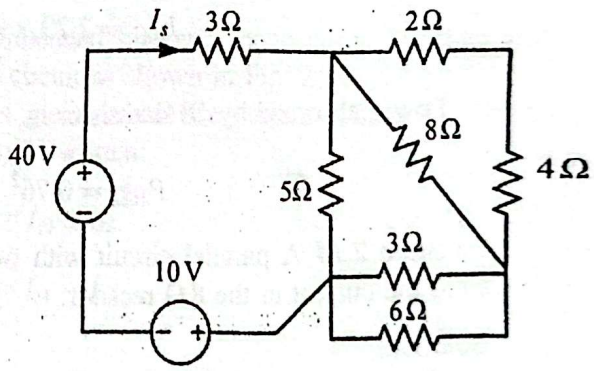


Fig. 2.42 Circuit for Practice Problem 2.13

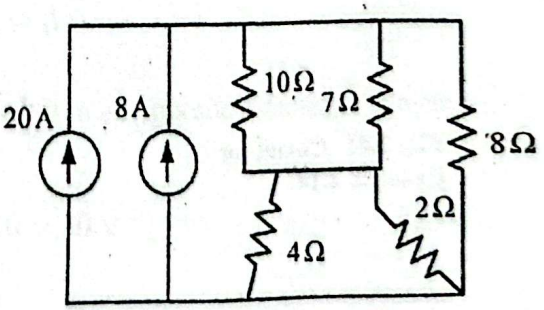


Fig. 2.43 Circuit for Practice Problem 2.14

2.11 Source Conversion Technique

Voltage and current sources are connected in an electrical circuit based on the application requirement. For circuit analysis, sometimes these sources need to be converted from one form to another to simplify the circuit configuration. This task can be performed using a source conversion technique. In this case, a resistance needs to be connected in series with a voltage source while a resistance needs to be connected in parallel with a current source. Figure 2.44 is considered to explain the source conversion technique.

From Fig. 2.44, the following expressions can be written:

$$I_s = \frac{V_s}{R} \tag{2.151}$$

$$V_s = I_s R \tag{2.152}$$

Example 2.15 A series–parallel circuit with voltage and current sources is shown in Fig. 2.45. Use source conversion technique to find the current in the 8 Ω resistor.

Solution:
The 30 V and 10 V voltage sources are converted to a current source as,

$$I_{s1} = \frac{30}{3} = 10\,\text{A} \tag{2.153}$$

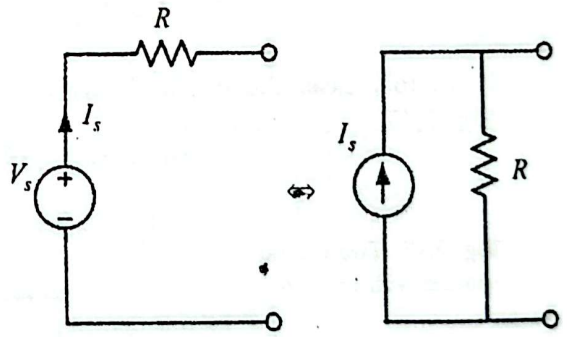


Fig. 2.44 Voltage and current sources with resistance

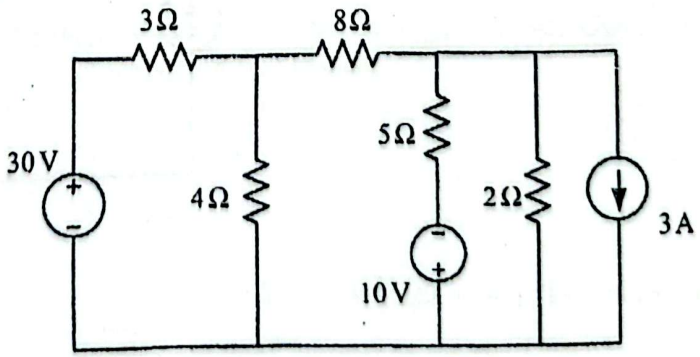


Fig. 2.45 Circuit for Example 2.15

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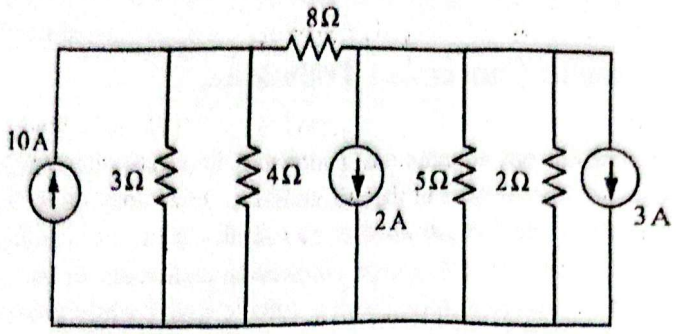


Fig. 2.46 Current sources with resistors

$$I_{s1} = \frac{10}{5} = 2\,\text{A} \tag{2.154}$$

The equivalent circuit is shown in Fig. 2.46. The resistors 3 Ω and 4 Ω, 5 Ω and 2 Ω are in parallel. Their equivalent resistances are,

$$R_1 = \frac{3 \times 4}{3+4} = 1.71\,\Omega \tag{2.155}$$

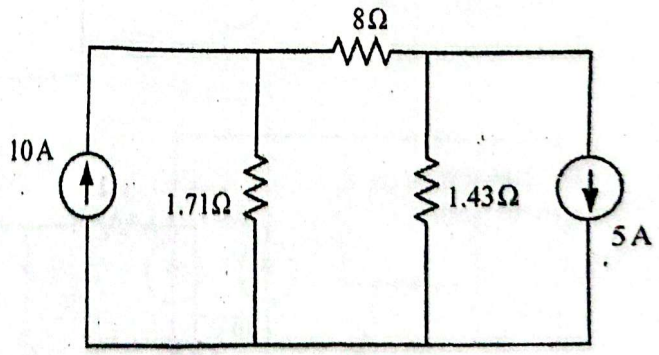
$$R_2 = \frac{5 \times 2}{5+2} = 1.43\,\Omega \tag{2.156}$$

The two current sources are added together and the value of the equivalent current source is,

$$I_s = 2+3 = 5\,\text{A} \tag{2.157}$$

In this case, the circuit is converted to an equivalent circuit as shown in Fig. 2.47.

Fig. 2.47 Two current sources with resistors



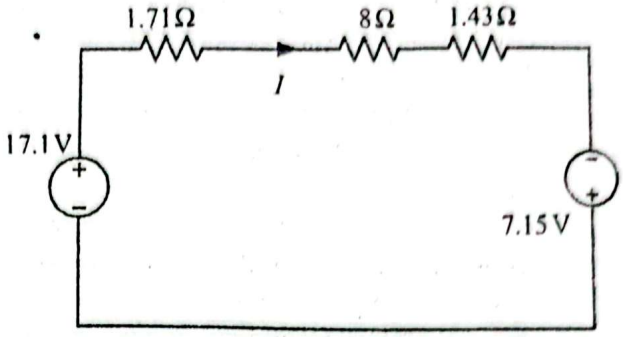


Fig. 2.48 Voltage sources with resistors

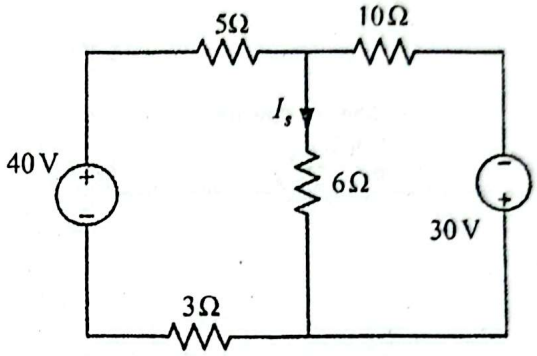


Fig. 2.49 Circuit for Practice Problem 2.15

Again, the current sources are converted to voltage sources as shown in Fig. 2.48, where the values of voltage sources are found to be,

$$V_{s1} = 10 \times 1.71 = 17.1\,\text{V} \tag{2.158}$$

$$V_{s2} = 5 \times 1.43 = 7.15\,\text{V} \tag{2.159}$$

Applying KVL to the circuit in Fig. 2.48 yields,

$$-17.1 + I(1.71 + 8 + 1.43) - 7.15 = 0 \tag{2.160}$$

$$I = \frac{24.25}{1.71 + 8 + 1.43} = 2.18\,\text{A} \tag{2.160}$$

Practice Problem 2.15

Figure 2.49 shows a series–parallel circuit with two voltage sources. Use source conversion technique to find the current in 6 Ω resistor.

Exercise Problems

2.1 A 750 W electric iron is connected to a 220 V voltage source. Determine the current drawn by the iron.

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2.2 A 15 Ω resistor is connected to a 240 V voltage source. Calculate the power absorbed by the resistor.

2.3 A circuit with different branches along with currents is shown in Fig. 2.50. Determine the values of the unknown currents.

2.4 Figure 2.51 shows a circuit with some unknown currents. Determine the unknown currents.

2.5 Some known and unknown voltages of a circuit are shown in Fig. 2.52. Use KVL to determine the unknown voltages.

2.6 Figure 2.53 shows a circuit with known and unknown voltages. Use KVL to find the voltages V_1 and V_2.

2.7 A voltage-controlled voltage source is shown in Fig. 2.54. Use KVL to determine the voltage V_x and the voltage between the points (nodes) a and b.

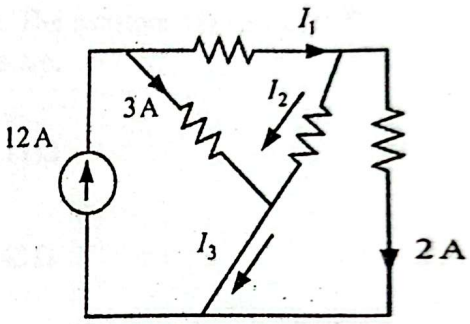


Fig. 2.50 Circuit for Problem 2.3

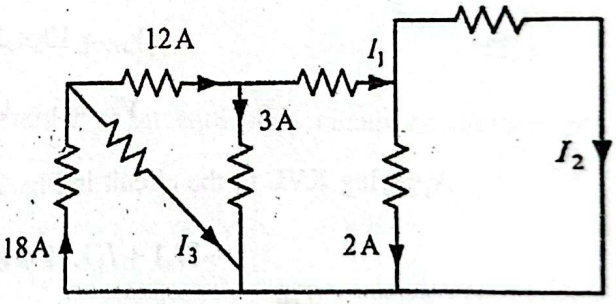


Fig. 2.51 Circuit for Problem 2.4

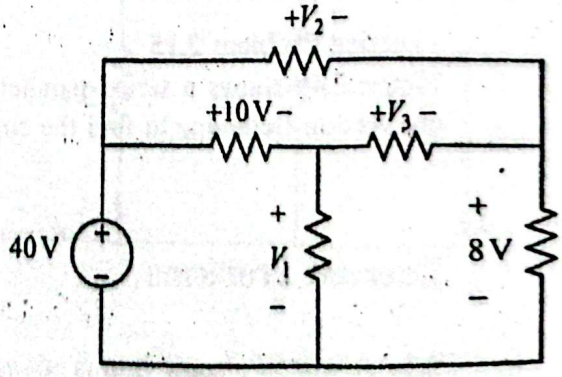


Fig. 2.52 Circuit for Problem 2.5

Fig. 2.53 Circuit for Problem 2.6

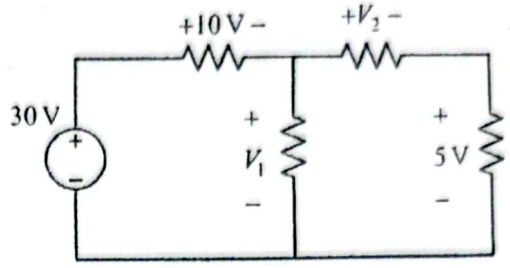


Fig. 2.54 Circuit for Problem 2.7

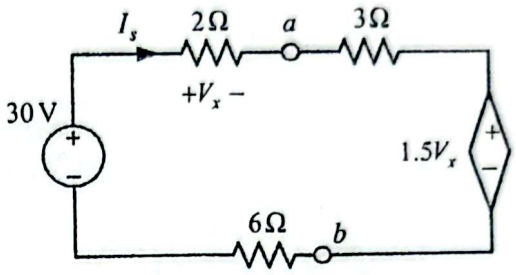


Fig. 2.55 Circuit for Problem 2.8

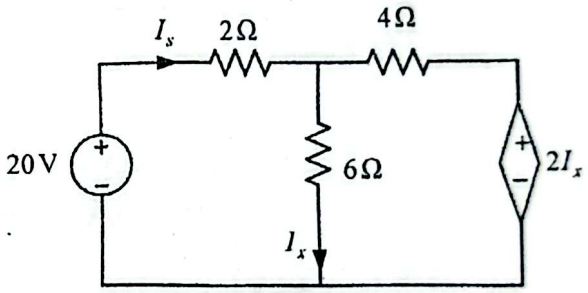
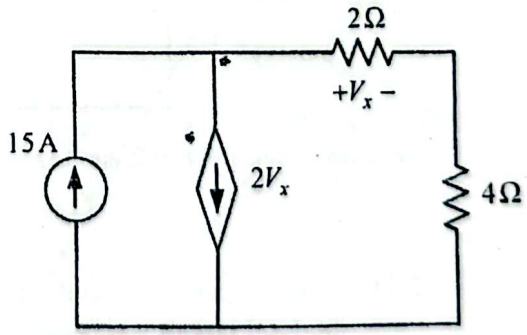


Fig. 2.56 Circuit for Problem 2.9



2.8 A current-controlled voltage source is shown in Fig. 2.55. Determine the currents I_s and I_x using KVL.

2.9 A voltage-controlled current source is shown in Fig. 2.56. Calculate the voltage drop across the 4 Ω resistor.

2.10 Figure 2.57 shows a current-controlled current source. Use KCL to determine the power absorbed by the 8 Ω resistor.

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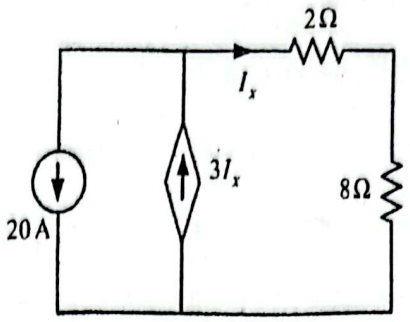


Fig. 2.57 Circuit for Problem 2.10

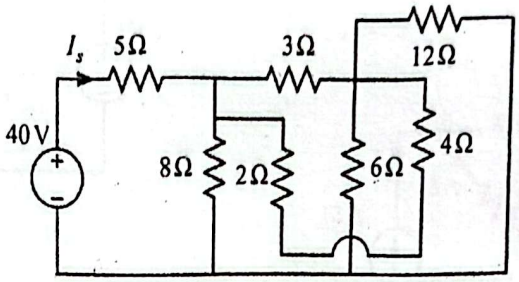


Fig. 2.58 Circuit for Problem 2.11

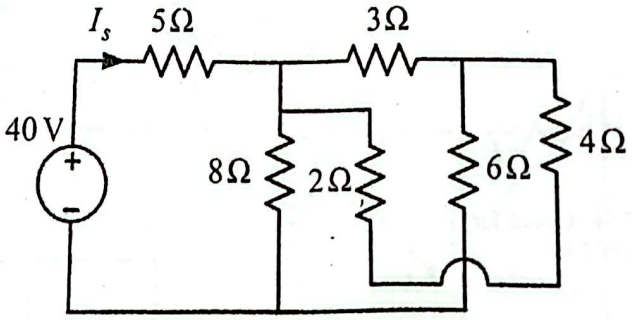


Fig. 2.59 Circuit for Problem 2.12

2.11 A series–parallel circuit with a voltage source is shown in Fig. 2.58. Calculate the total circuit resistance and the source current.

2.12 Figure 2.59 shows a series–parallel circuit with a voltage source. Calculate the total circuit resistance and the source current.

2.13 A series–parallel circuit with a voltage source is shown in Fig. 2.60. Determine the total circuit resistance and the source current.

2.14 A series–parallel circuit is shown in Fig. 2.61. Calculate the total circuit resistance and the power absorbed by the 4 Ω resistor.

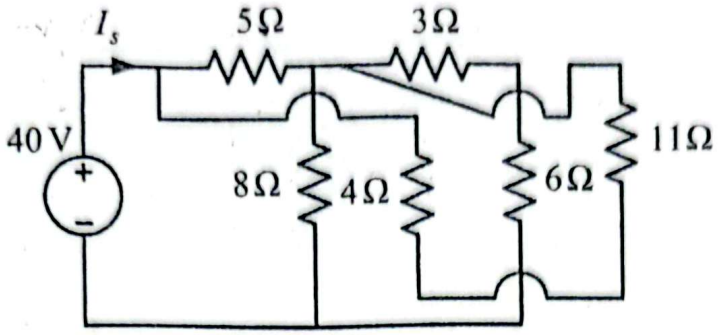


Fig. 2.60 Circuit for Problem 2.13

Fig. 2.61 Circuit for Problem 2.14

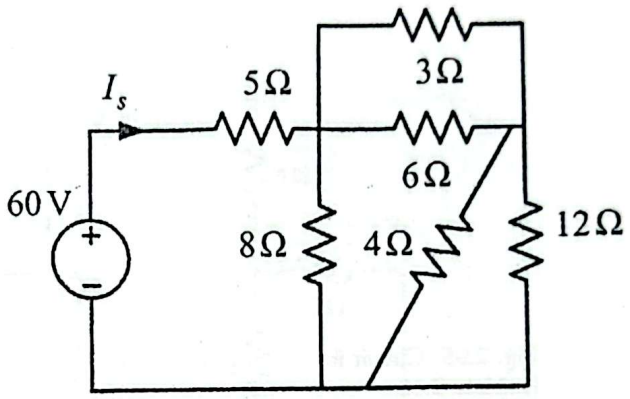
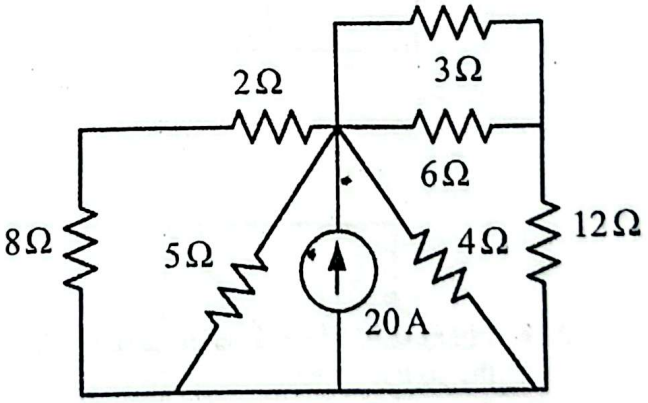


Fig. 2.62 Circuit for Problem 2.15



2.15 Find the current through the 4 Ω resistor of the circuit shown in Fig. 2.62.

2.16 A series–parallel circuit is shown in Fig. 2.63. Calculate the source current and the voltage drop across the 4 Ω resistor.

2.17 Figure 2.64 shows a series–parallel circuit. Determine the source current and the voltage drop across the 8 Ω resistor.

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Fig. 2.63 Circuit for Problem 2.16

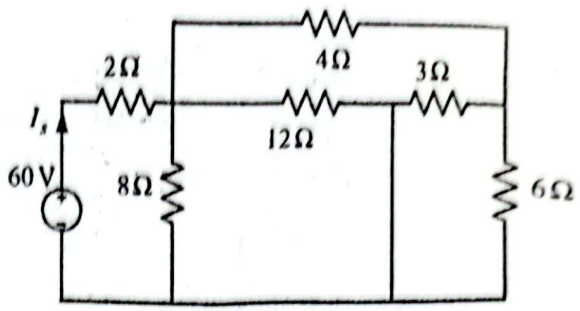


Fig. 2.64 Circuit for Problem 2.17

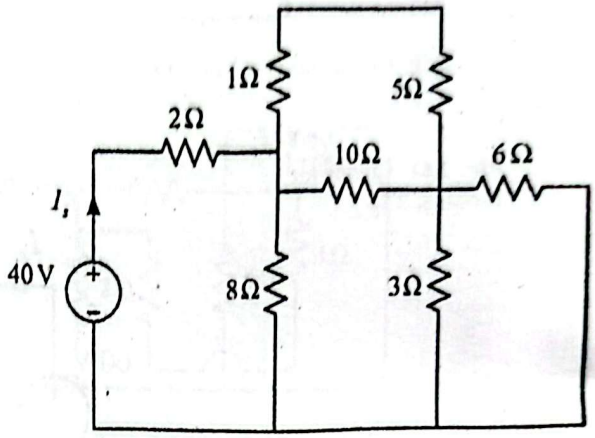
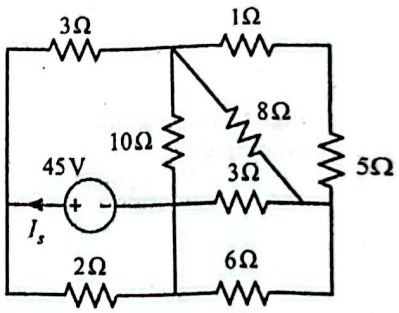


Fig. 2.65 Circuit for Problem 2.18



2.18 Figure 2.65 shows a series–parallel circuit. Find the total circuit resistance and the source current.

2.19 A series–parallel circuit is shown in Fig. 2.66. Calculate the total circuit resistance and the source current.

2.20 Figure 2.67 shows a series–parallel circuit. Calculate the total circuit resistance and the source current.

2.21 A series–parallel electrical circuit is shown in Fig. 2.68. Find the total circuit resistance and the source current.

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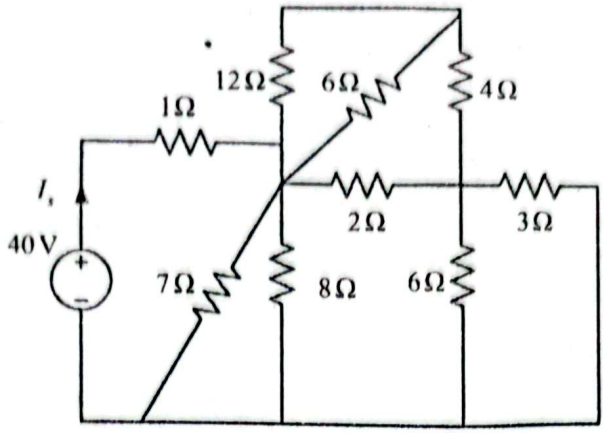


Fig. 2.66 Circuit for Problem 2.19

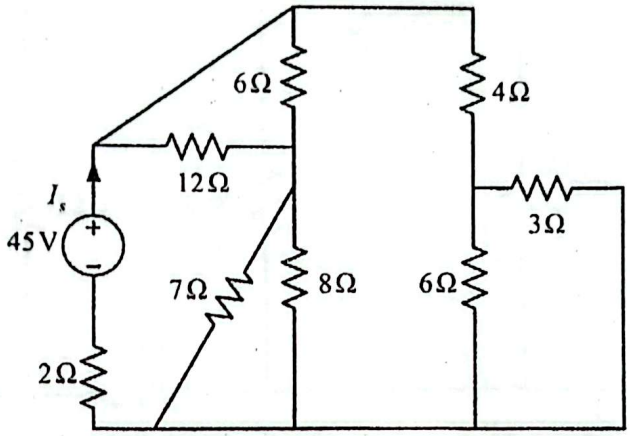


Fig. 2.67 Circuit for Problem 2.20

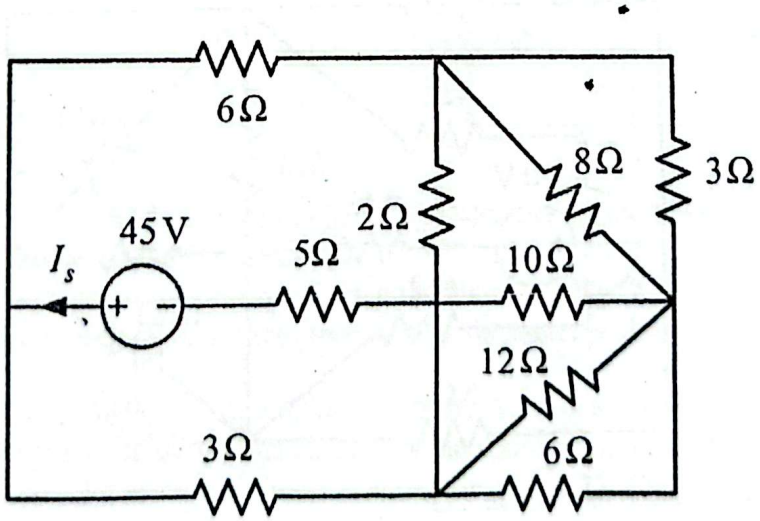


Fig. 2.68 Circuit for Problem 2.21

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2.22 Figure 2.69 shows a series–parallel electrical circuit. Calculate the total circuit resistance and the source current.

2.23 A series–parallel circuit is shown in Fig. 2.70. Determine the total circuit resistance and the source current.

2.24 A series–parallel circuit is shown in Fig. 2.71. Calculate the total circuit resistance and the source current.

2.25 An electrical circuit is shown in Fig. 2.72. Determine the total circuit resistance and the power absorbed by the 2 Ω resistor.

2.26 Figure 2.73 shows an electrical circuit. Calculate the total circuit resistance and the current through the 5 Ω resistor.

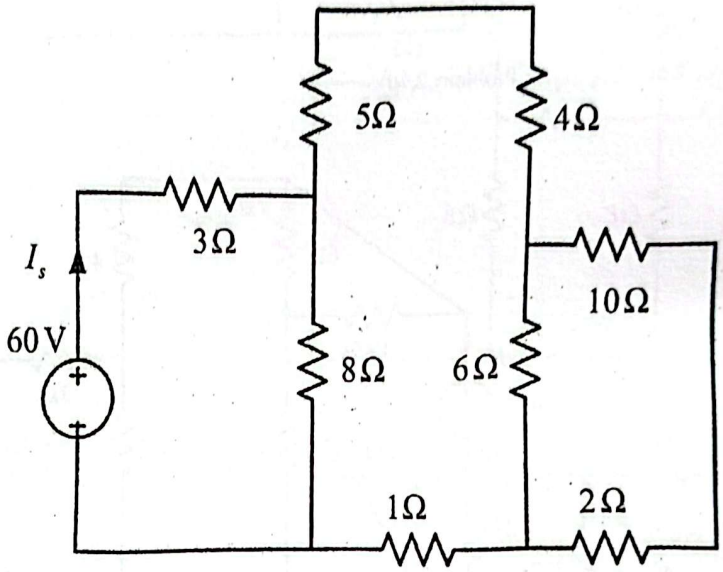


Fig. 2.69 Circuit for Problem 2.22

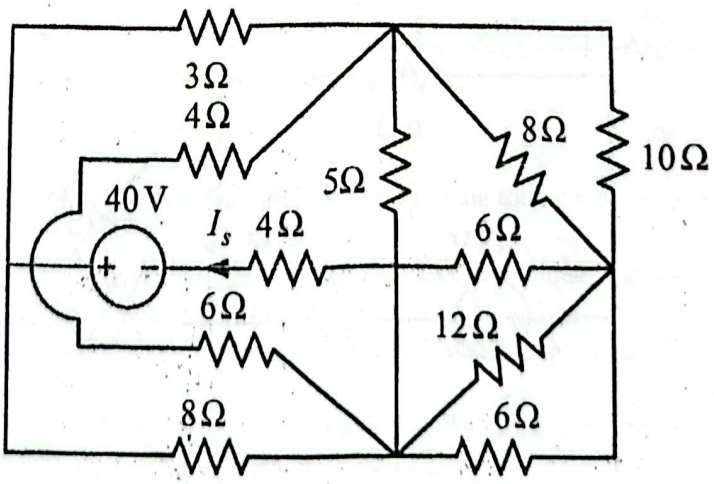


Fig. 2.70 Circuit for Problem 2.23

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Fig. 2.71 Circuit for Problem 2.24

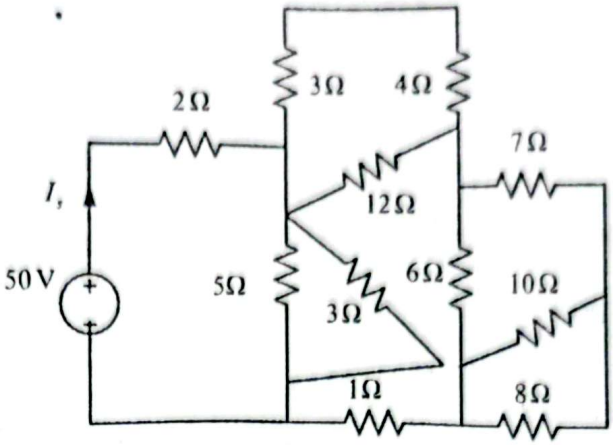


Fig. 2.72 Circuit for Problem 2.25

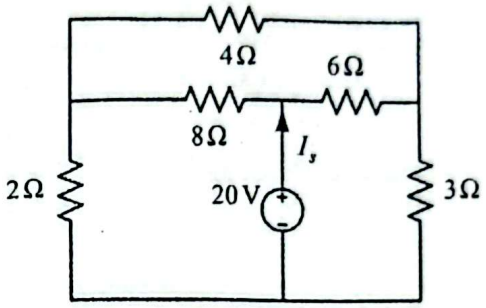
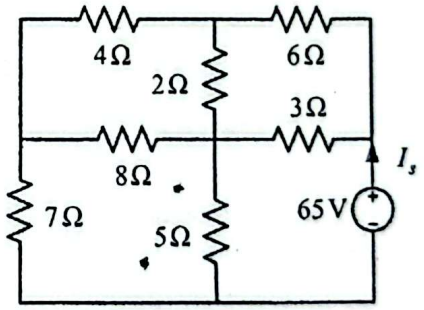


Fig. 2.73 Circuit for Problem 2.26



2.27 An electrical circuit is shown in Fig. 2.74. Calculate the total circuit resistance and the source current in the circuit.

2.28 A delta–wye-connected electrical circuit is shown in Fig. 2.75. Determine the total circuit resistance, source current and the voltage drop across the 3 Ω resistor.

2.29 An electrical circuit is shown in Fig. 2.76. Use delta–wye conversion to determine the total circuit resistance and the source current in the circuit.

2.30 A series–parallel electrical circuit is shown in Fig. 2.77. Calculate the power absorbed by the 4 Ω resistor.

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Fig. 2.74 Circuit for Problem 2.27

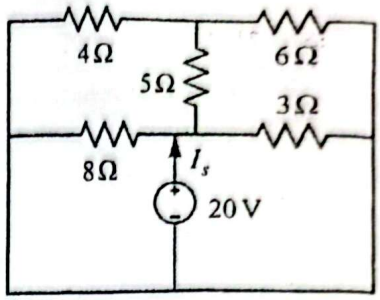


Fig. 2.75 Circuit for Problem 2.28

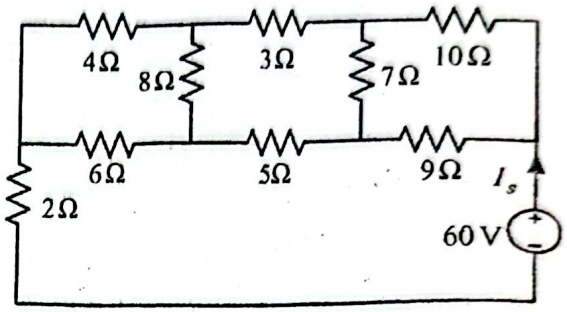


Fig. 2.76 Circuit for Problem 2.29

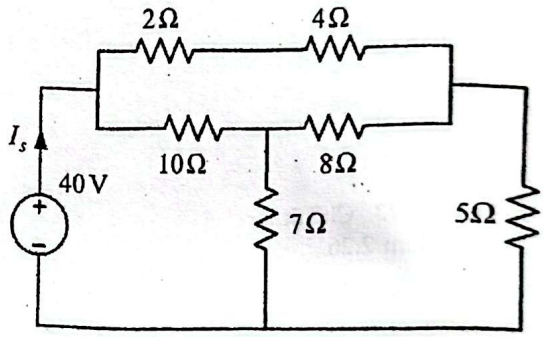
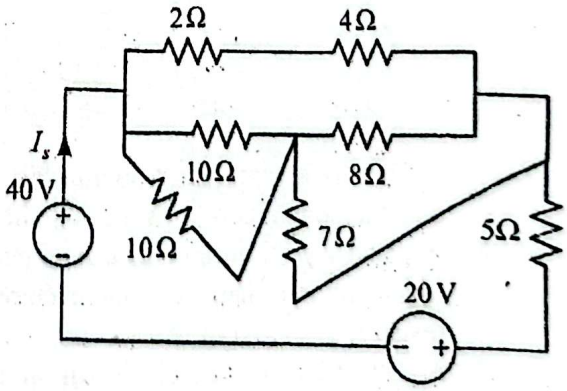


Fig. 2.77 Circuit for Problem 2.30



2.31 Figure 2.78 shows a series–parallel electrical circuit. Calculate the current in the 4 Ω resistor.

2.32 Figure 2.79 shows a series–parallel electrical circuit. Calculate the value of the voltage V_x.

2.33 A series–parallel electrical circuit with two current sources is shown in Fig. 2.80. Determine the current in the 5 Ω resistor.

2.34 A series–parallel electrical circuit is shown in Fig. 2.81. Calculate the voltage drop across the 4 Ω resistor.

2.35 Figure 2.82 shows a series–parallel electrical circuit. Determine the power absorbed by the 10 Ω resistor.

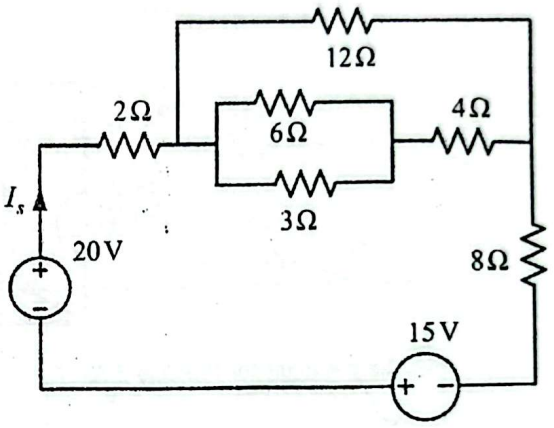


Fig. 2.78 Circuit for Problem 2.31

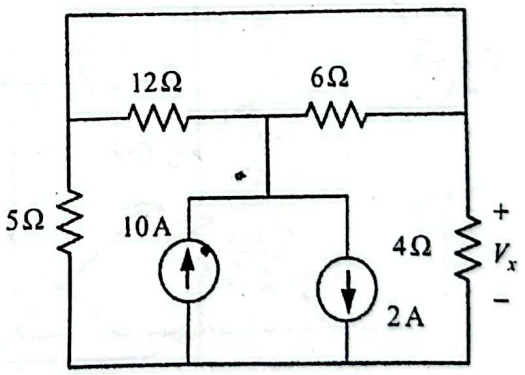


Fig. 2.79 Circuit for Problem 2.32

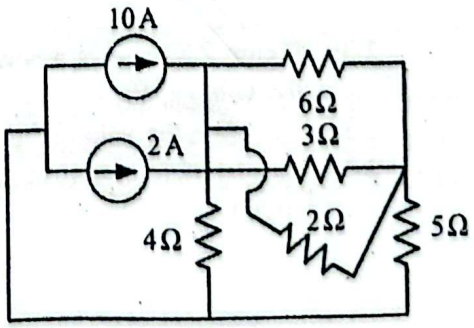


Fig. 2.80 Circuit for Problem 2.33

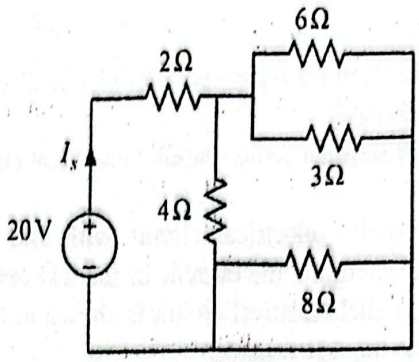


Fig. 2.81 Circuit for Problem 2.34

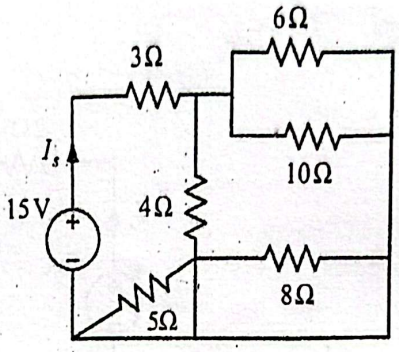


Fig. 2.82 Circuit for Problem 2.35

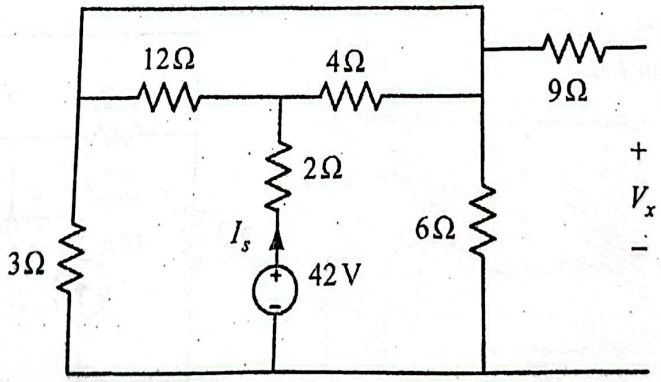


Fig. 2.83 Circuit for Problem 2.36

2.36 Figure 2.83 shows a series–parallel electrical circuit. Determine the value of the voltage, V_x.

2.37 Calculate the value of the voltage, V_0 of the circuit as shown in Fig. 2.84.

2.38 An electrical circuit is shown in Fig. 2.85. Calculate the voltage, V_0, of the circuit.

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Fig. 2.84 Circuit for Problem 2.37

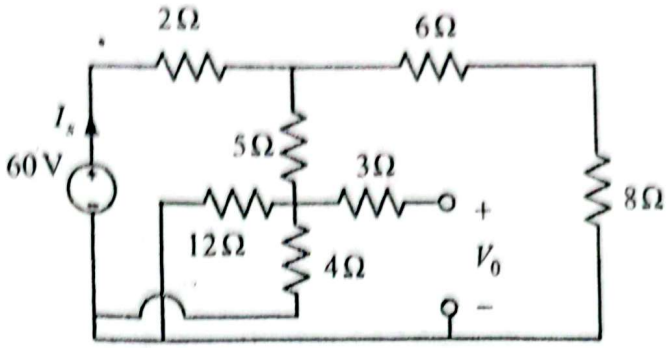


Fig. 2.85 Circuit for Problem 2.38

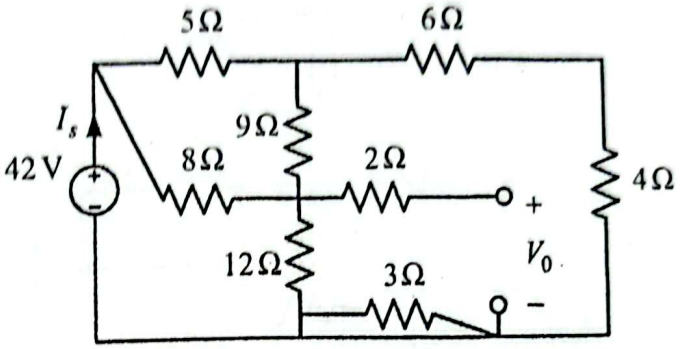
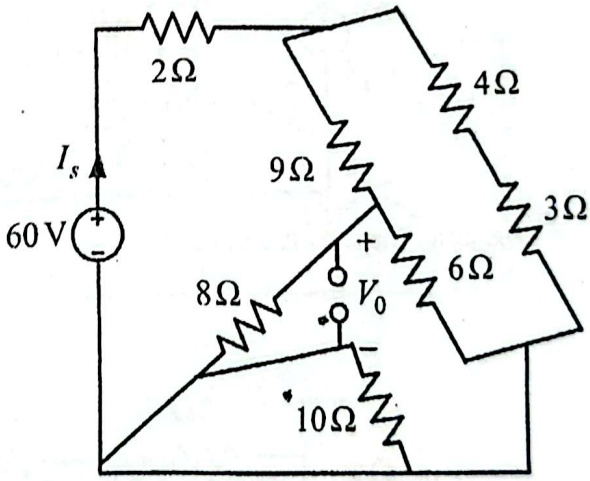


Fig. 2.86 Circuit for Problem 2.39



2.39 An electrical circuit is shown in Fig. 2.86. Determine the voltage, V_0, of the circuit.

2.40 Figure 2.87 shows an electrical circuit. Calculate the voltage, V_0, of the circuit.

2.41 An electrical circuit is shown in Fig. 2.88. Find the voltage, V_0, of the circuit.

2.42 An electrical circuit is shown in Fig. 2.89. Determine the voltage, V_0, of the circuit.

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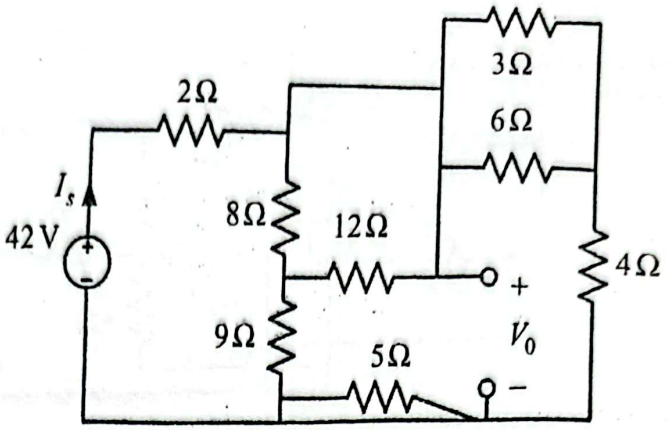


Fig. 2.87 Circuit for Problem 2.40

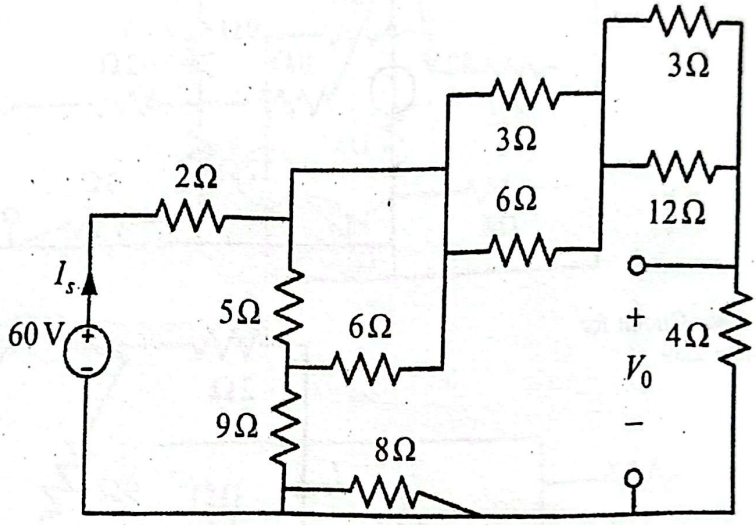


Fig. 2.88 Circuit for Problem 2.41

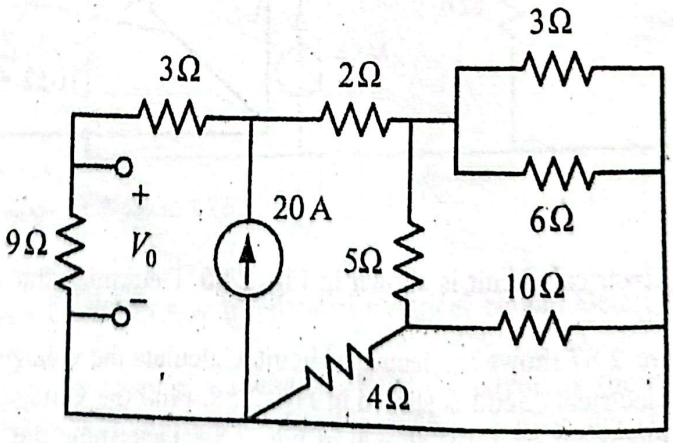


Fig. 2.89 Circuit for Problem 2.42

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2.43 An electrical circuit is shown in Fig. 2.90. Calculate the voltage, V_0, of the circuit.

2.44 Figure 2.91 shows an electrical circuit. Determine the voltage, V_x, of the circuit.

2.45 An electrical circuit is shown in Fig. 2.92. Calculate the voltage, V_0, of the circuit.

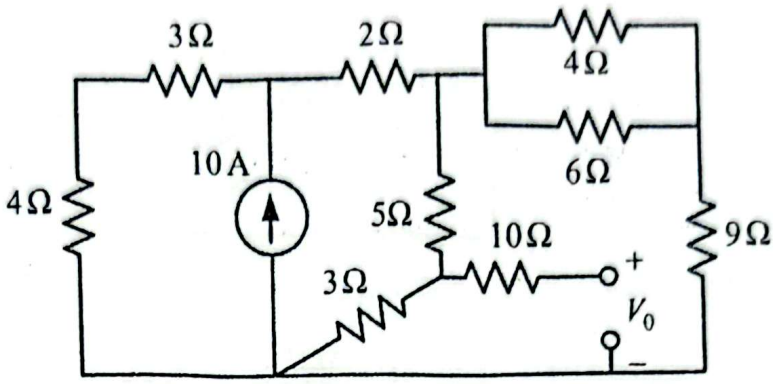


Fig. 2.90 Circuit for Problem 2.43

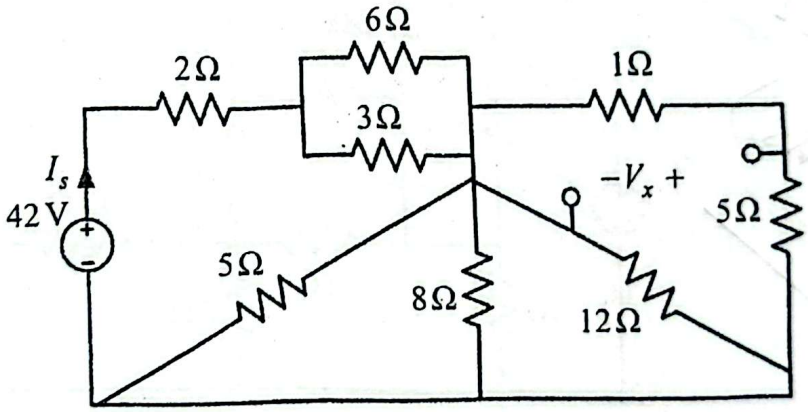


Fig. 2.91 Circuit for Problem 2.44

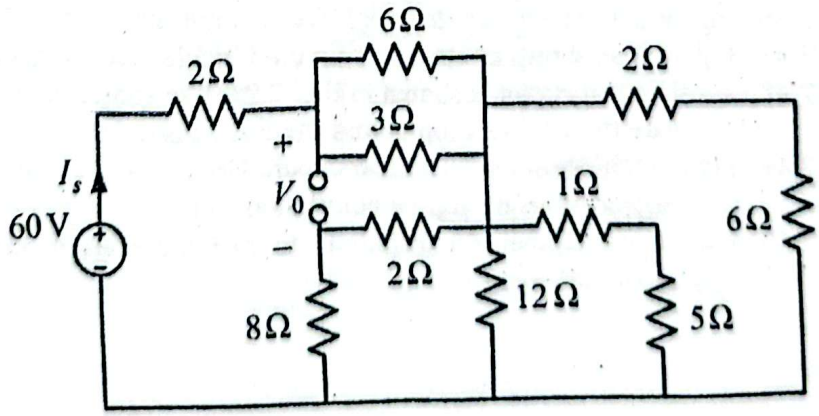


Fig. 2.92 Circuit for Problem 2.45

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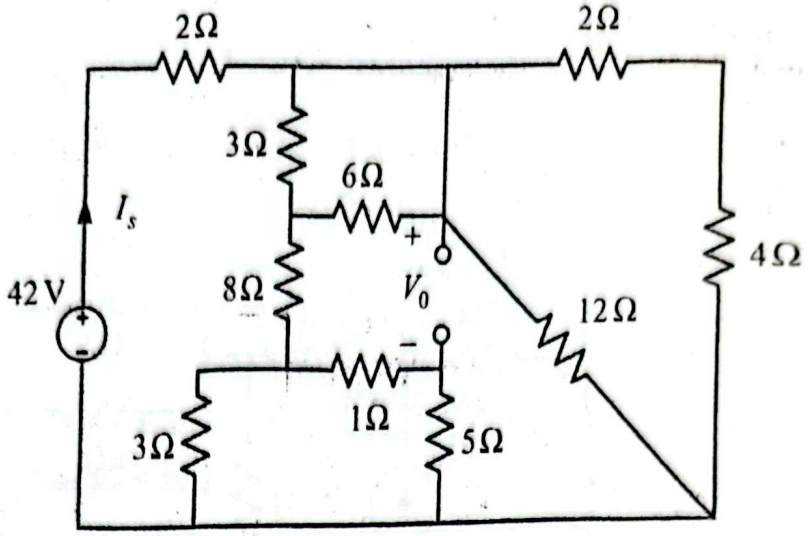
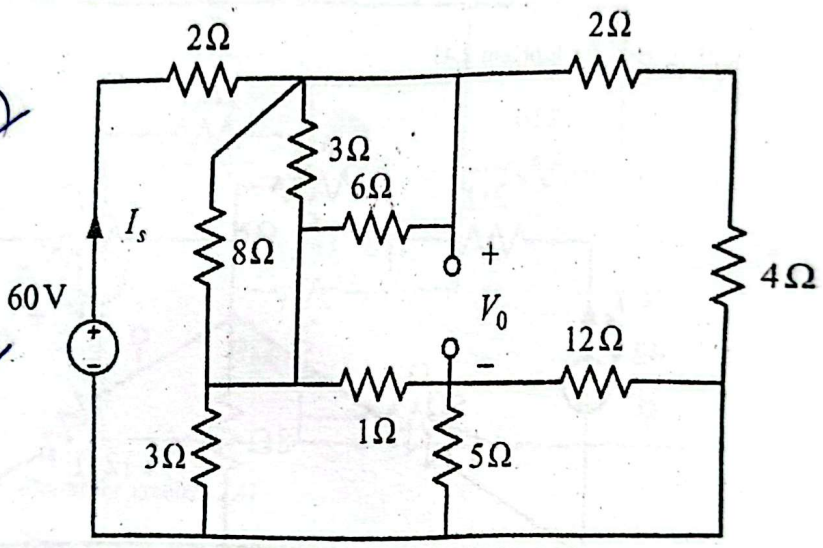


Fig. 2.93 Circuit for Problem 2.46



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(6 Marks)

Fig. 2.94 Circuit for Problem 2.47

2.46 Figure 2.93 shows an electrical circuit. Find the voltage, V_0, of the circuit.

2.47 Figure 2.94 shows an electrical circuit. Calculate the voltage, V_0, of the circuit.

2.48 An electrical circuit is shown in Fig. 2.95. Use source conversion technique to calculate the voltage drop across the 5 Ω resistor.

2.49 Figure 2.96 shows an electrical circuit. Determine the voltage drop across the 2 Ω resistor by using source conversion technique.

2.50 Use source conversion technique to find the voltage drop across the 3 Ω resistor of the circuit in Fig. 2.97.

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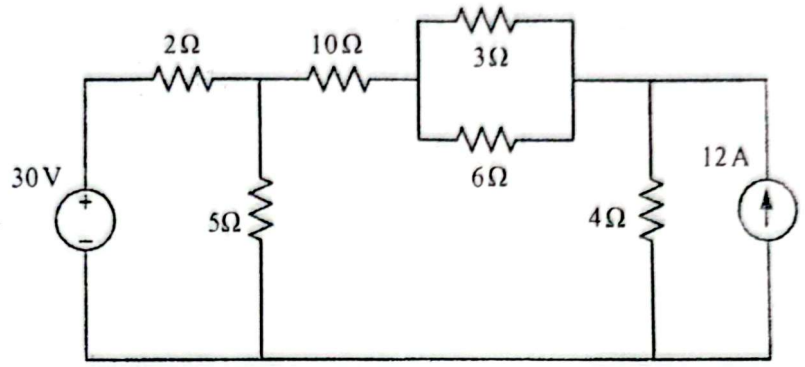


Fig. 2.95 Circuit for Problem 2.48

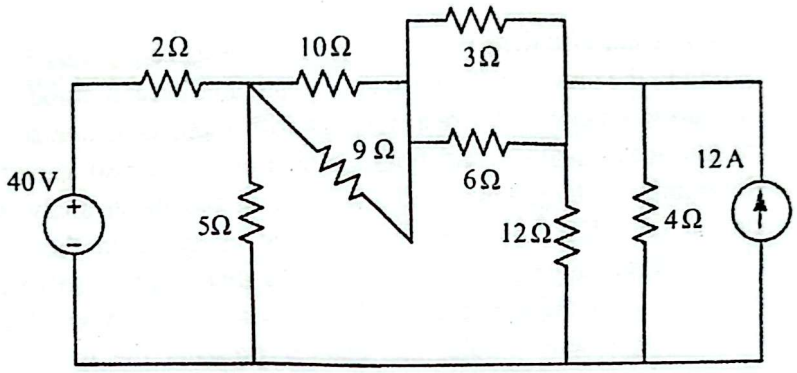
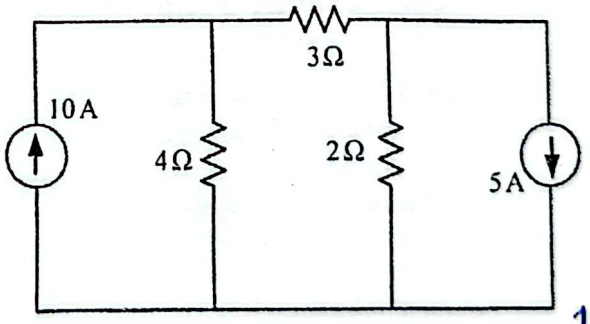


Fig. 2.96 Circuit for Problem 2.49

Fig. 2.97 Circuit for Problem 2.50



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Chapter 3
Different Methods for Circuit Analysis

3.1 Introduction

Fundamental electrical laws and the associated parameters have already been discussed in the previous chapters. Additional analytical methods have been developed to analyse practical electrical circuits with more than one source. These methods include mesh analysis and node voltage analysis, also known as nodal analysis. Kirchhoff's voltage law (KVL) is used in the mesh analysis while Kirchhoff's current law (KCL) is used in the nodal analysis. In these analytical methods, Cramer's rule appears to be a handy tool that helps to solve the system of linear equations after the application of Kirchhoff's laws in an electrical circuit. Upon introducing Cramer's rule, this chapter presents the theoretical foundations on supermesh, supernode, mesh and nodal analysis with dependent and independent sources.

3.2 Cramer's Rule

In 1750, Gabriel Cramer developed an algebraic rule for solving unknown parameters. According to his name, it is known as Cramer's rule. It is an efficient rule for a system of two or more equations. Consider the three linear equations as follows:

$$a_1x + b_1y + c_1z = d_1 \tag{3.1}$$

$$a_2x + b_2y + c_2z = d_2 \tag{3.2}$$

$$a_3x + b_3y + c_3z = d_3 \tag{3.3}$$

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The determinant of those equations is,

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \tag{3.4}$$

The values of the three unknown parameters x, y and z can be determined as,

$$x = \frac{D_x}{D} \tag{3.5}$$

$$y = \frac{D_y}{D} \tag{3.6}$$

$$z = \frac{D_z}{D} \tag{3.7}$$

The values of D_x, D_y and D_z are determined as,

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \tag{3.8}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \tag{3.9}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \tag{3.10}$$

Example 3.1 Three simultaneous equations of currents of a multisource electrical circuit are given by $2I_1 + 3I_2 + 6I_3 = 5$, $3I_1 + 5I_2 - 9I_3 = 8$, $4I_1 + 6I_2 + 8I_3 = 10$. Determine the unknown currents.

Solution:

The determinant of three simultaneous equations is,

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 3 & 5 & -9 \\ 4 & 6 & 8 \end{vmatrix} = 2(40+54) - 3(24+36) + 6(18-20) \tag{3.11}$$

$$= -4$$

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Other related parameters can be determined as,

$$D_1 = \begin{vmatrix} 5 & 3 & 6 \\ 8 & 5 & -9 \\ 10 & 6 & 8 \end{vmatrix} = 5(40+54) - 3(64+90) + 6(48-50) \quad (3.12)$$
$$= -4$$

$$D_2 = \begin{vmatrix} 2 & 5 & 6 \\ 3 & 8 & -9 \\ 4 & 10 & 8 \end{vmatrix} = 2(64+90) - 5(24+36) + 6(30-32) \quad (3.13)$$
$$= -4$$

$$D_3 = \begin{vmatrix} 2 & 3 & 5 \\ 3 & 5 & 8 \\ 4 & 6 & 10 \end{vmatrix} = 2(50-48) - 3(30-32) + 5(18-20) \quad (3.14)$$
$$= 0$$

The values of the currents can be determined as,

$$I_1 = \frac{D_1}{D} = \frac{-4}{-4} = 1\,\text{A} \quad (3.15)$$

$$I_2 = \frac{D_2}{D} = \frac{-4}{-4} = 1\,\text{A} \quad (3.16)$$

$$I_3 = \frac{D_3}{D} = \frac{0}{-4} = 0\,\text{A} \quad (3.17)$$

Practice Problem 3.1 Three simultaneous equations of currents of a multisource electrical circuit are given by $I_1 + 3I_2 - 4I_3 = 5$, $2I_1 + 3I_2 - 4I_3 = 5$, $3I_1 + 2I_2 + 5I_3 = 7$. Calculate the unknown currents.

3.3 Mesh Analysis with Independent Source

An electrical circuit with an independent voltage sources is shown in Fig. 3.1. In mesh analysis, the current sources are required to be converted to voltage sources with appropriate polarity as identified by the current directions of the current sources. Applying KVL to the circuit in Fig. 3.1 yields,

$$-V_1 + I_1R_1 + R_3(I_1 - I_2) + I_1R_2 = 0 \quad (3.18)$$

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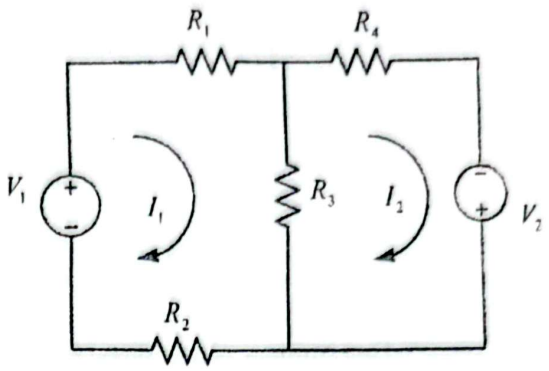


Fig. 3.1 Circuit with two voltage sources

$$I_1(R_1 + R_3 + R_2) - I_2R_3 = V_1 \tag{3.19}$$

$$-V_2 + R_3(I_2 - I_1) + I_2R_4 = 0 \tag{3.20}$$

$$-I_1R_3 + (R_3 + R_4)I_2 = V_2 \tag{3.21}$$

Equation (3.21) can be written as,

$$I_1 = \frac{R_3 + R_4}{R_3} I_2 - \frac{V_2}{R_3} \tag{3.22}$$

Substituting Eq. (3.22) into Eq. (3.19) yields,

$$\left(\frac{R_3 + R_4}{R_3} I_2 - \frac{V_2}{R_3}\right)(R_1 + R_3 + R_2) - I_2R_3 = V_1 \tag{3.23}$$

Equations (3.22) and (3.23) can be used to determine the mesh currents.

Example 3.2 Determine the power absorbed by the 6 Ω resistor using mesh analysis for the circuit in Fig. 3.2.

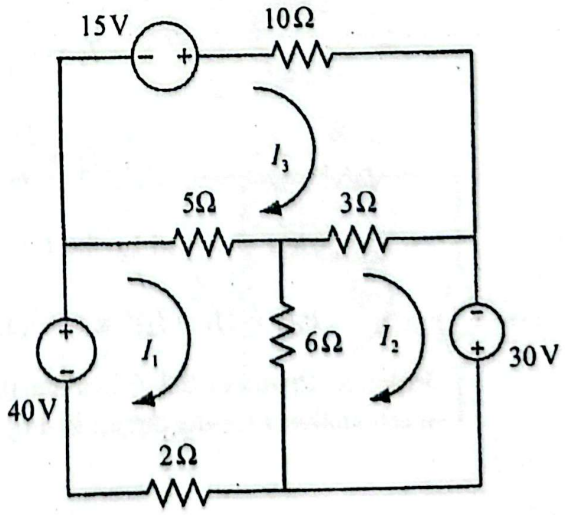


Fig. 3.2 Circuit for Example 3.2

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Solution:
Applying KVL to the meshes 1, 2 and 3 yields,

$$13I_1 - 6I_2 - 5I_3 = 40 \tag{3.24}$$

$$-6I_1 + 9I_2 - 3I_3 = 30 \tag{3.25}$$

$$-2I_1 + 3I_2 - I_3 = 10 \tag{3.26}$$

$$-5I_1 - 3I_2 + 18I_3 = 15 \tag{3.27}$$

Determinant of the three simultaneous equations is calculated as,

$$D = \begin{vmatrix} 13 & -6 & -5 \\ -2 & 3 & -1 \\ -5 & -3 & 18 \end{vmatrix} = 13(54-3) + 6(-36-5) - 5(6+15) \tag{3.28}$$
$$= 312$$

Other related parameters can be determined as,

$$D_1 = \begin{vmatrix} 40 & -6 & -5 \\ 10 & 3 & -1 \\ 15 & -3 & 18 \end{vmatrix} = 40(54-3) + 6(180+15) - 5(-30-45) \tag{3.29}$$
$$= 3585$$

$$D_2 = \begin{vmatrix} 13 & 40 & -5 \\ -2 & 10 & -1 \\ -5 & 15 & 18 \end{vmatrix} = 13(180+15) - 40(-36-5) - 5(-30+50) \tag{3.30}$$
$$= 4075$$

The values of the currents are calculated as,

$$I_1 = \frac{D_1}{D} = \frac{3585}{312} = 11.49 \text{ A} \tag{3.31}$$

$$I_2 = \frac{D_2}{D} = \frac{4075}{312} = 13.06 \text{ A} \tag{3.32}$$

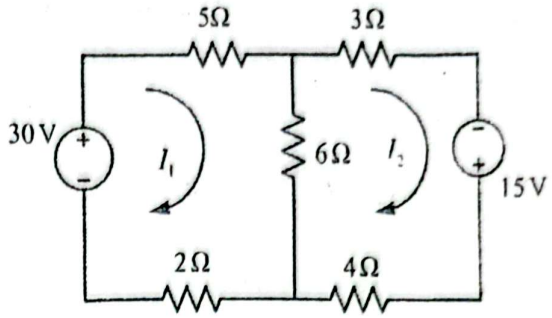
The power absorbed by the 6 Ω resistor is,

$$P_{6\Omega} = (I_1 - I_2)^2 \times 6 = (11.49 - 13.06)^2 \times 6 = 14.79 \text{ W} \tag{3.33}$$

Practice Problem 3.2 Calculate the voltage drop across the 4 Ω resistor using mesh analysis for the circuit in Fig. 3.3.

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Fig. 3.3 Circuit for Practice Problem 3.2



3.4 Mesh Analysis with Dependent Source

An electrical circuit with a dependent source is shown in Fig. 3.4. Here, voltage-controlled voltage source is considered for mesh analysis. Based on the assigned directions of mesh currents as shown in Fig. 3.4, the application of KVL yields,

$$(R_1 + R_2 + R_4)I_1 - R_2I_2 = V_1 \tag{3.34}$$

$$-R_2I_1 + (R_2 + R_3)I_2 = mV_x \tag{3.35}$$

According to the circuit configuration,

$$V_x - I_1R_1 = 0 \tag{3.36}$$

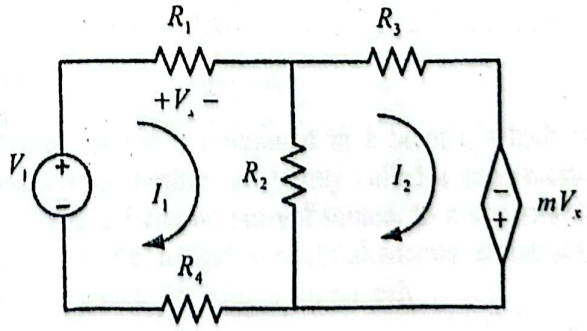
$$V_x = I_1R_1 \tag{3.37}$$

Substituting Eq. (3.37) into Eq. (3.35) yields,

$$-R_2I_1 + (R_2 + R_3)I_2 = I_1R_1 \tag{3.38}$$

$$-(R_1 + R_2)I_1 + (R_2 + R_3)I_2 = 0 \tag{3.39}$$

Fig. 3.4 Circuit mesh analysis with a dependent source



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The determinant can be determined as,

$$D = \begin{vmatrix} R_1 + R_2 + R_4 & -R_2 \\ -(R_1 + R_2) & R_2 + R_3 \end{vmatrix} = (R_1 + R_2 + R_4)(R_2 + R_3) - (R_1 + R_2)R_2 \quad (3.4$$

According to Cramer's rule, the other parameters can be written as,

$$D_1 = \begin{vmatrix} V_1 & -R_2 \\ 0 & R_2 + R_3 \end{vmatrix} = V_1(R_2 + R_3) \quad (3.4$$

$$D_2 = \begin{vmatrix} R_1 + R_2 + R_4 & V_1 \\ -(R_1 + R_2) & 0 \end{vmatrix} = (R_1 + R_2)V_1 \quad (3.4$$

Then mesh currents can be determined as,

$$I_1 = \frac{D_1}{D} = \frac{V_1(R_2 + R_3)}{(R_1 + R_2 + R_4)(R_2 + R_3) - (R_1 + R_2)R_2} \quad (3.4$$

$$I_2 = \frac{D_2}{D} = \frac{V_1(R_1 + R_2)}{(R_1 + R_2 + R_4)(R_2 + R_3) - (R_1 + R_2)R_2} \quad (3.4$$

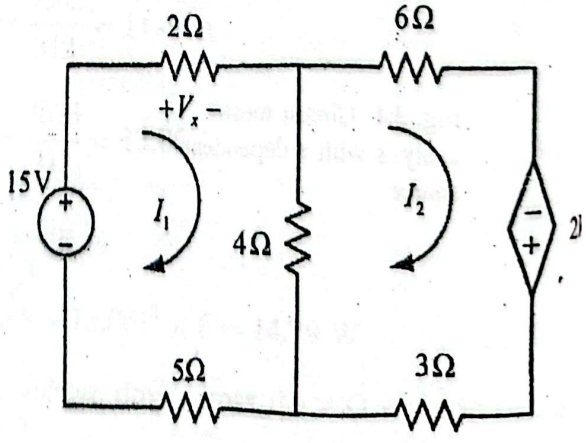
Example 3.3 Determine the power absorbed by the 6 Ω resistor and voltage dr across the 4 Ω resistor using mesh analysis for the circuit in Fig. 3.5. Apply PSpi simulation to find the mesh currents.

Solution:
Applying KVL to the circuit in Fig. 3.5 yields,

$$11I_1 - 4I_2 = 15 \quad (3.4$$

$$-4I_1 + 13I_2 = 2V_x \quad (3.4$$

Fig. 3.5 Circuit for Example 3.3



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According to the given circuit,

$$V_x = 2I_1 \tag{3.47}$$

Substituting Eq. (3.47) into Eq. (3.47) yields,

$$-4I_1 + 13I_2 = 4I_1 \tag{3.48}$$

$$I_1 = \frac{13I_2}{8} = 1.63I_2 \tag{3.49}$$

Substituting Eq. (3.49) into Eq. (3.45) yields,

$$11(1.63I_2) - 4I_2 = 15 \tag{3.50}$$

$$I_2 = \frac{15}{13.93} = 1.08 \text{ A} \tag{3.51}$$

Substituting Eq. (3.51) into Eq. (3.49) yields,

$$I_1 = 1.63 \times 1.08 = 1.76 \text{ A} \tag{3.52}$$

The power absorbed by the 6 Ω resistor is,

$$P_{6\,\Omega} = I_2^2 \times 6 = 1.08^2 \times 6 = 7 \text{ W} \tag{3.53}$$

The voltage drop across the 4 Ω resistor is,

$$V_{4\,\Omega} = 4 \times (I_1 - I_2) = 4 \times (1.76 - 1.08) = 2.72 \text{ V} \tag{3.54}$$

The PSpice simulation circuit is shown in Fig. 3.6. The simulation mesh currents are the same as the calculated currents.

Practice Problem 3.3 Calculate the mesh currents in the circuit shown in Fig. 3.7. Verify the results by the PSpice simulation.

3.5 Supermesh Circuit

In an electric circuit, when a current source is contained in a branch, which is common between two meshes, these two meshes are jointly called a supermesh. The supermesh is treated in a way as if there is no current source. In a supermesh circuit, both KCL and KVL are applied to get the simultaneous equations. Figure 3.8 shows an electrical circuit, which contains a supermesh.

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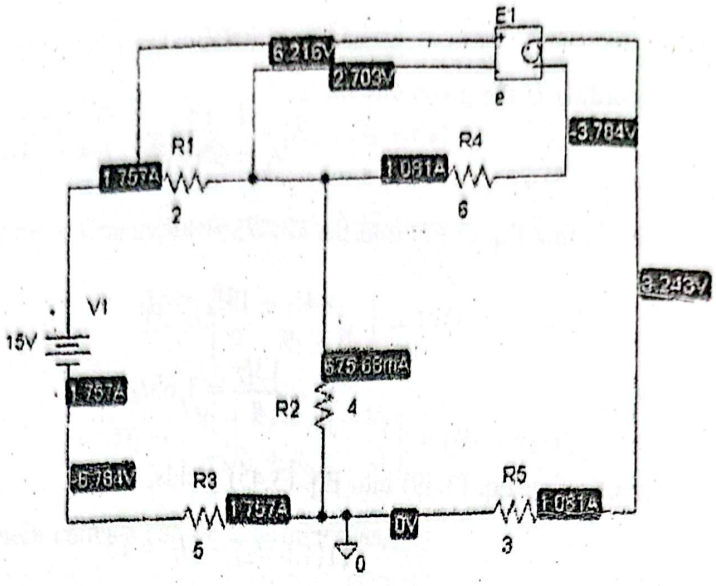


Fig. 3.6 PSpice simulation circuit for Example 3.3

Fig. 3.7 Circuit for Practice Problem 3.3

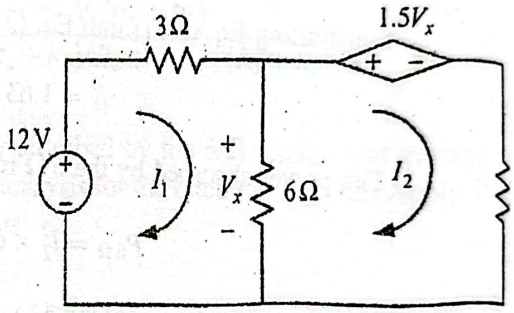
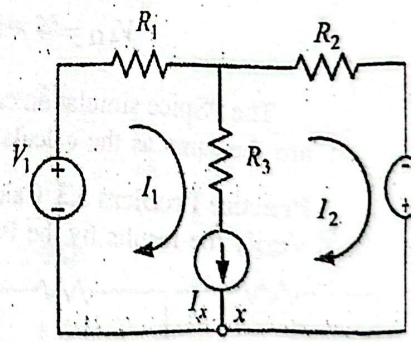


Fig. 3.8 Circuit with a common current source



Applying KCL at the node x yields,

$$I_1 = I_x + I_2 \quad (3.$$

$$I_x = I_1 - I_2 \quad (3.$$

Now in the supermesh, after removing the current source as shown in Fig. 3.9, the applying of KVL yields,

$$-V_1 + I_1R_1 + I_2R_2 - V_2 = 0 \tag{3.57}$$

$$I_1R_1 + I_2R_2 = V_1 + V_2 \tag{3.58}$$

The mesh currents can be calculated from Eqs. (3.56) and (3.58), if the other electrical quantities are known.

Example 3.4 Find the power absorbed by the 6 Ω resistor and voltage drop across the 1 Ω resistor using mesh analysis for the circuit in Fig. 3.10. Apply PSpice simulation to find the mesh currents.

Solution:
Applying KCL at the node x yields,

$$I_1 = 4 + I_2 \tag{3.59}$$

The supermesh circuit is shown in Fig. 3.11. Applying KVL to the circuit in Fig. 3.11 yields,

$$-12 + 3I_1 + 10I_2 - 16 = 0 \tag{3.60}$$

$$3I_1 + 10I_2 = 28 \tag{3.61}$$

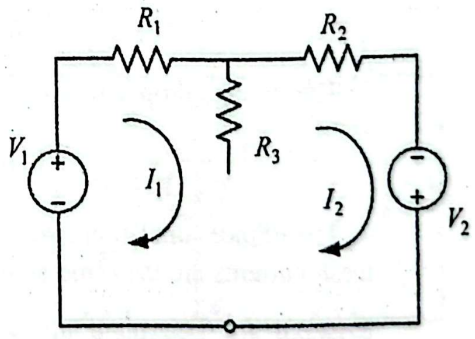


Fig. 3.9 A supermesh circuit

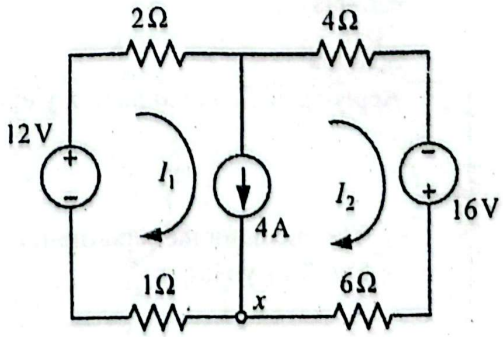


Fig. 3.10 A circuit for Example 3.4

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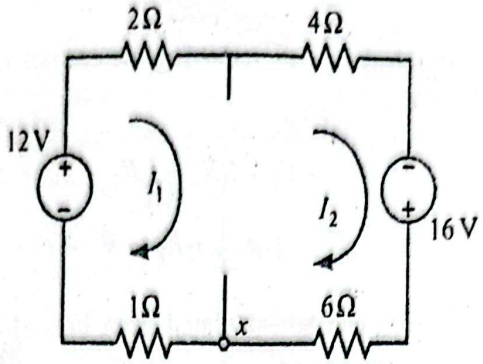


Fig. 3.11 Supermesh circuit for Example 3.4

Substituting Eq. (3.59) into Eq. (3.61) yields,

$$3(4+I_2)+10I_2=28 \tag{3.62}$$

$$13I_2=16 \tag{3.63}$$

$$I_2=\frac{16}{13}=1.23\text{ A} \tag{3.64}$$

Substituting Eq. (3.63) into Eq. (3.59) yields,

$$I_1=4+1.23=5.23\text{ A} \tag{3.65}$$

The power absorbed by the 6 Ω resistor is,

$$P_{6\Omega}=1.23^2\times 6=9.08\text{ W} \tag{3.66}$$

The voltage drop across the 1 Ω resistor is,

$$V_{1\Omega}=5.23\times 1=5.23\text{ V} \tag{3.67}$$

The PSpice simulation circuit is shown in Fig. 3.12. The values of the calculated mesh currents are the same as the simulation mesh currents.

Example 3.5 Determine the voltage drop across the 2 Ω resistor using mesh analysis for the circuit in Fig. 3.13. Apply PSpice simulation to find the mesh currents.

Solution:
Applying KCL at the node x yields,

$$I_1=4+I_2 \tag{3.68}$$

The circuit for the supermesh is shown in Fig. 3.14. Applying KVL to the circuit in Fig. 3.14 yields,

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$$-16 + 6I_1 + 10I_2 + 1.5V_x = 0 \tag{3.68}$$

According to the given circuit,

$$V_x = 4I_1 \tag{3.69}$$

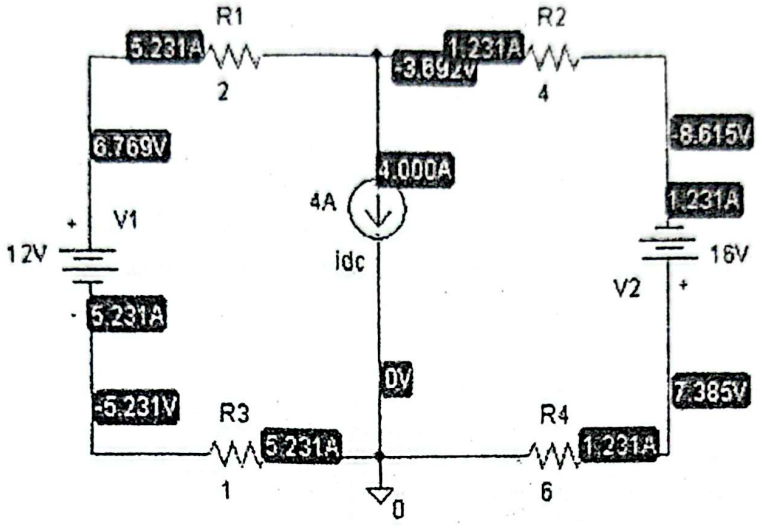


Fig. 3.12 Simulation circuit for Example 3.4

Fig. 3.13 A circuit for Example 3.5

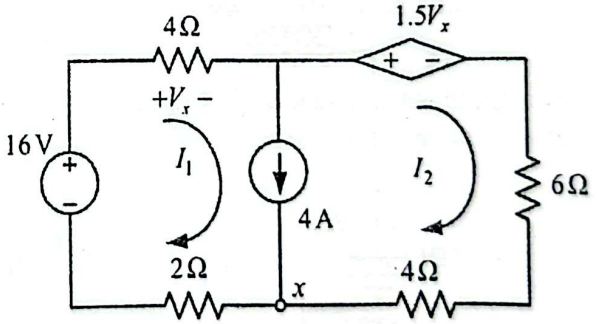
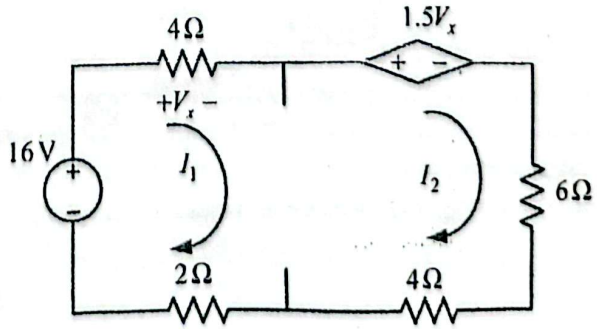


Fig. 3.14 A supermesh circuit for Example 3.5



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Substituting Eqs. (3.67) and (3.69) into Eq. (3.68) yields,

$$-16 + 6I_1 + 10I_2 + 1.5 \times 4I_1 = 0 \tag{3.7}$$

$$12(4 + I_2) + 10I_2 = 16 \tag{3.7}$$

$$I_2 = \frac{-32}{22} = -1.45 \text{ A} \tag{3.7}$$

Substituting Eq. (3.72) into Eq. (3.67) yields,

$$I_1 = 4 - 1.45 = 2.55 \text{ A} \tag{3.7}$$

The voltage drop across the 2 Ω resistor is,

$$V_{2\Omega} = 2 \times 2.3 = 4.6 \text{ V} \tag{3.7}$$

The simulation circuit is shown in Fig. 3.15. The values of the simulation mesh currents are the same as the calculated mesh currents.

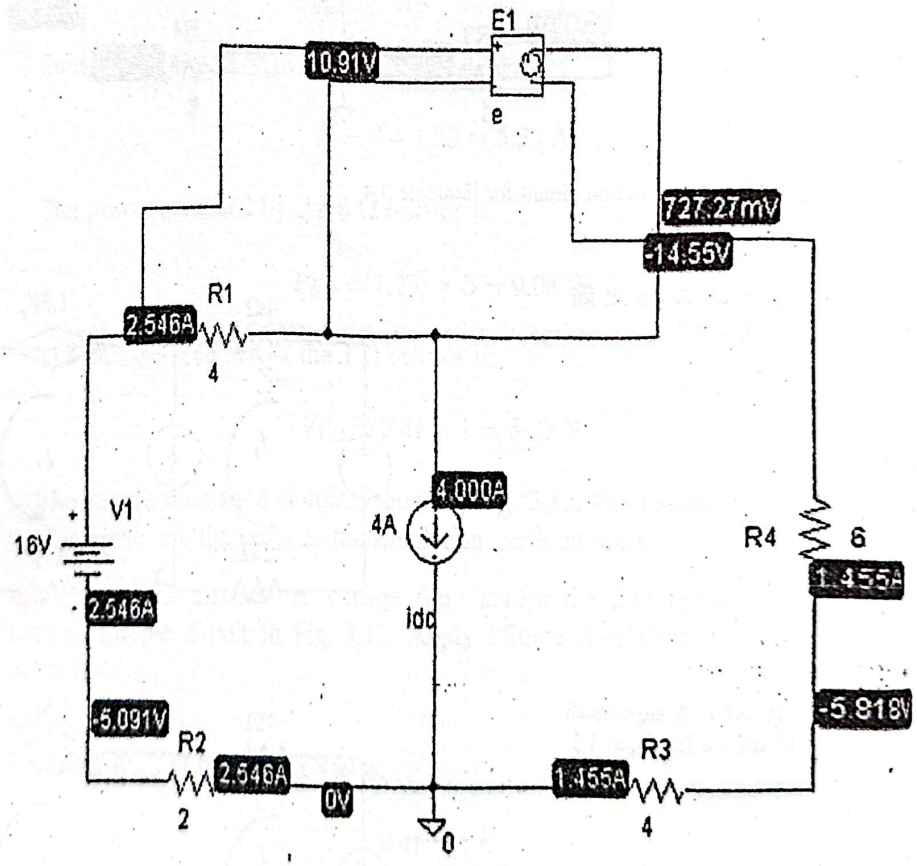


Fig. 3.15 Simulation circuit for Example 3.5

Fig. 3.16 A circuit for Practice Problem 3.4

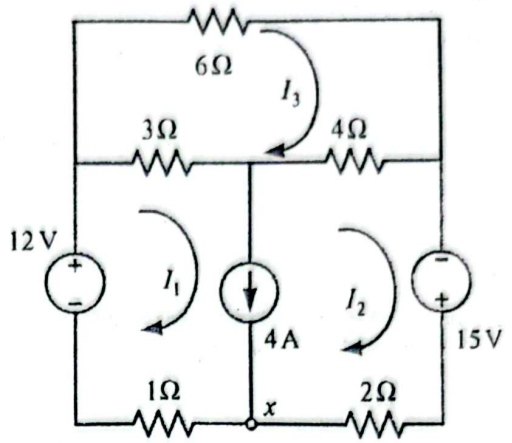
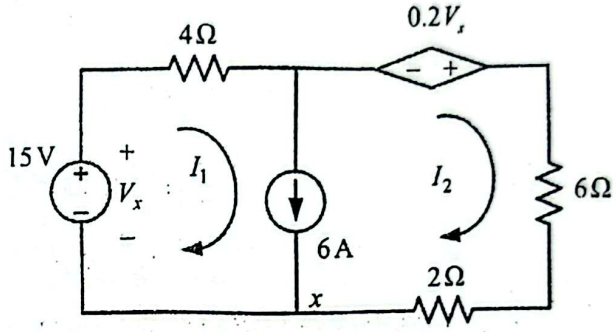


Fig. 3.17 A circuit for Practice Problem 3.5



Practice Problem 3.4 Calculate the power absorbed by the 3 Ω resistor using mesh analysis of the circuit in Fig. 3.16. Use PSpice simulation to find the mesh currents.

Practice Problem 3.5 Find the power absorbed by the 6 Ω resistor using mesh analysis for the circuit in Fig. 3.17. Use PSpice simulation to find the mesh currents.

3.6 Nodal Analysis

Nodal analysis is another method to calculate the electrical parameters such as current, voltage and power of a circuit that contains more than one source. In this method, reference and non-reference nodes need to be identified.

Then, if possible, any voltage source needs to be converted to a current source. Finally, KCL is applied in each node to get the simultaneous equations, which are solved to calculate the electrical parameters. Figure 3.18 shows an electrical circuit with voltage and current sources. A voltage source is converted to a current source. A circuit with reference ($V = 0$) and non-reference nodes (V_1, V_2) is shown in Fig. 3.19. Then, applying KCL at nodes 1 (V_1) and 2 (V_2) yields,

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Fig. 3.18 First circuit for nodal analysis

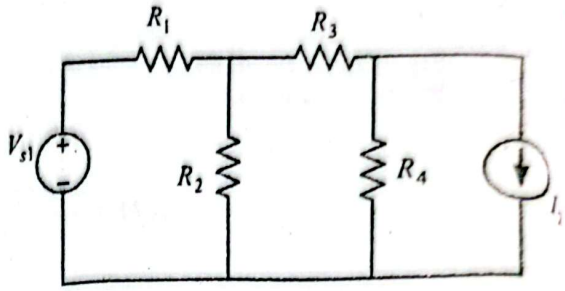
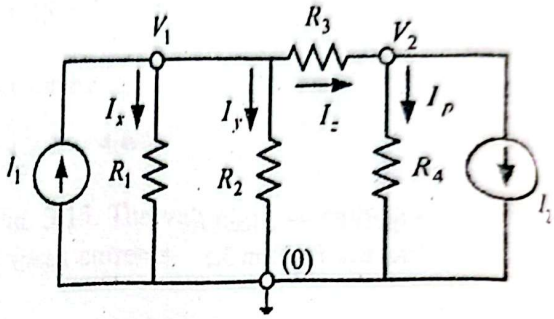


Fig. 3.19 Second circuit for nodal analysis



$$I_1 = I_x + I_y + I_z \tag{3.75}$$

$$I_z = I_p + I_2 \tag{3.76}$$

Equations (3.75) and (3.76) can be expanded in terms of node voltages as,

$$I_1 = \frac{V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} \tag{3.77}$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_1 - \frac{1}{R_3}V_2 = I_1 \tag{3.78}$$

$$\frac{V_1 - V_2}{R_3} = \frac{V_2}{R_4} + I_2 \tag{3.79}$$

$$\frac{1}{R_3}V_1 - \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_2 = I_2 \tag{3.80}$$

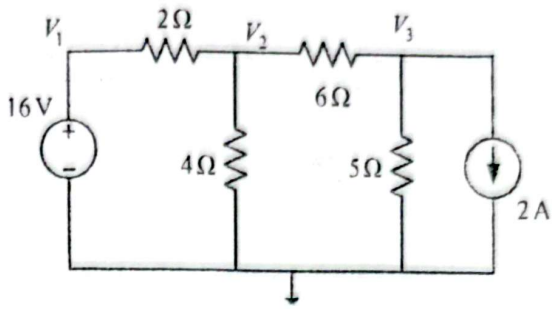
The values of node voltages can be calculated from Eqs. (3.78) and (3.80) by any methods if the other electrical quantities are known.

Example 3.6 Find the power absorbed by the 6 Ω resistor using nodal analysis of the circuit in Fig. 3.20. Use PSpice simulation to find the node voltages.

Solution:
Applying KCL at nodes 1, 2 and 3 yields,

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Fig. 3.20 An electrical circuit for Example 3.6



$$V_1 = 16\,\text{V} \tag{3.81}$$

$$\frac{V_1 - V_2}{2} = \frac{V_2}{4} + \frac{V_2 - V_3}{6} \tag{3.82}$$

$$6V_1 - 6V_2 = 3V_2 + 2V_2 - 2V_3 \tag{3.83}$$

$$11V_2 - 2V_3 = 96 \tag{3.84}$$

$$\frac{V_2 - V_3}{6} = \frac{V_3}{5} + 2 \tag{3.85}$$

$$5V_2 - 5V_3 = 6V_3 + 60 \tag{3.86}$$

$$5V_2 - 11V_3 = 60 \tag{3.87}$$

From Eqs. (3.84) and (3.87), the node voltage, V_3 can be determined as,

$$V_3 = \frac{-180}{111} = -1.62\,\text{V} \tag{3.88}$$

The value of the node voltage, V_2 can be determined as,

$$V_2 = \frac{60 + 11 \times (1.62)}{5} = 8.44\,\text{V} \tag{3.89}$$

The current through the 6 Ω resistor is calculated as,

$$I_{6\,\Omega} = \frac{8.44 - (-1.62)}{6} = 1.68\,\text{A} \tag{3.90}$$

The power absorbed by the 6 Ω resistor is calculated as,

$$P_{6\,\Omega} = 1.68^2 \times 6 = 16.93\,\text{W} \tag{3.91}$$

The PSpice simulation circuit is shown in Fig. 3.21. The simulation node voltages are the same as the calculated node voltages.

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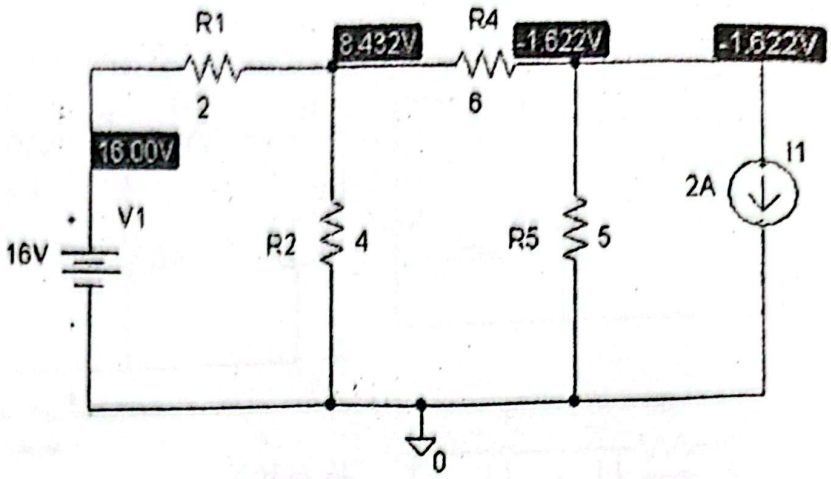
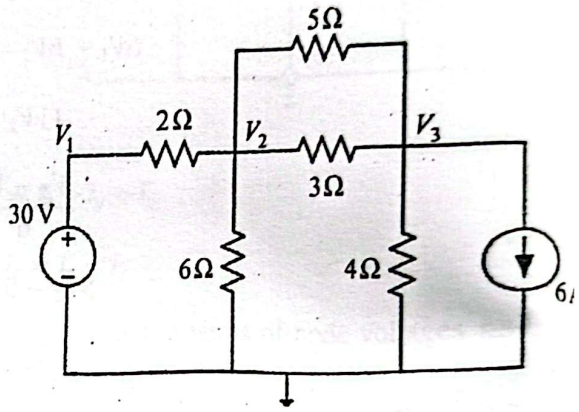


Fig. 3.21 PSpice simulation circuit for Example 3.6

Fig. 3.22 Circuit for Practice Problem 3.6



Practice Problem 3.6 Determine the power absorbed by the 6Ω resistor using nodal analysis for the circuit in Fig. 3.22. Use PSpice simulation to find the node voltages.

3.7 Nodal Analysis with Dependent Source

Sometimes, an electrical circuit may contain dependent source such as the one shown in Fig. 3.23 with current-controlled current source. In this circuit, applying KCL at node 1 yields,

$$I_{s1} = I_x + I_y \tag{3.92}$$

$$I_{s1} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \tag{3.93}$$

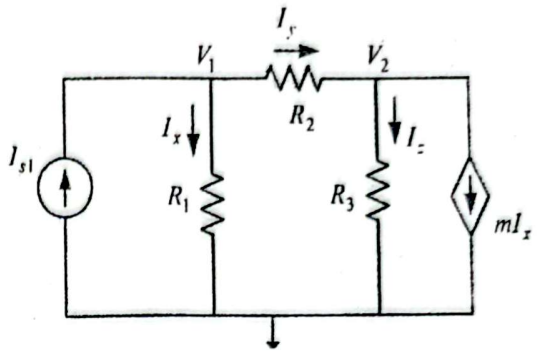


Fig. 3.23 Circuit with dependent current source for nodal analysis

$$I_{s1} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \frac{1}{R_2}V_2 \tag{3.94}$$

$$I_{s1} = (G_1 + G_2)V_1 - G_2V_2 \tag{3.95}$$

Applying KCL at node 2 yields,

$$I_y = I_z + mI_x \tag{3.96}$$

$$\frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3} + m\frac{V_1}{R_1} \tag{3.97}$$

$$0 = \left(\frac{1}{R_2} - m\frac{1}{R_1}\right)V_1 - \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_2 \tag{3.98}$$

$$0 = (G_2 - mG_1)V_1 - (G_2 + G_3)V_2 \tag{3.99}$$

From Eqs. (3.95) and (3.99), the values of node voltages can be determined if other electrical quantities are given.

Example 3.7 Calculate the current in the 6Ω resistor using nodal analysis for the circuit in Fig. 3.24. Use PSpice simulation to find the node voltages.

Solution:
Applying KCL at node 1 yields,

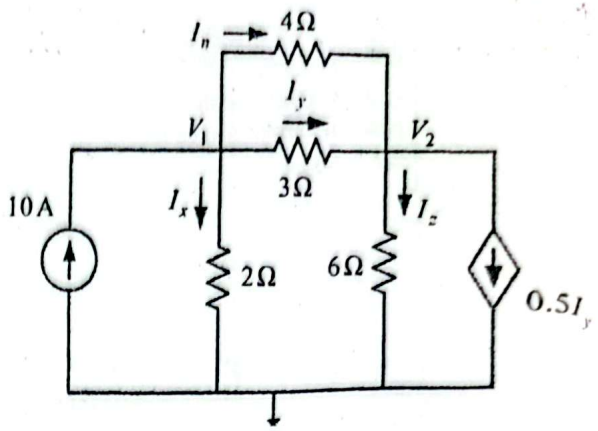
$$10 = I_x + I_y + I_n \tag{3.100}$$

$$10 = \frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{4} \tag{3.101}$$

$$120 = 6V_1 + 4V_1 - 4V_2 + 3V_1 - 3V_2 \tag{3.102}$$

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Fig. 3.24 Circuit for Example 3.7



$$13V_1 - 7V_2 = 120 \tag{3.103}$$

Applying KCL at node 2 yields,

$$I_y + I_n = I_z + 0.5I_y \tag{3.104}$$

$$0.5I_y + I_n = I_z \tag{3.105}$$

$$0.5\frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{4} = \frac{V_2}{6} \tag{3.106}$$

$$2V_1 - 2V_2 + 3V_1 - 3V_2 = 2V_2 \tag{3.107}$$

$$V_1 = \frac{7}{5}V_2 \tag{3.108}$$

Substituting Eq. (3.108) into Eq. (3.103) yields,

$$13\frac{7}{5}V_2 - 7V_2 = 120 \tag{3.109}$$

$$V_2 = \frac{600}{56} = 10.71 \text{ V} \tag{3.110}$$

The current in the 6 Ω resistor is,

$$I_z = \frac{V_2}{6} = \frac{10.71}{6} = 1.79 \text{ A} \tag{3.111}$$

The PSpice simulation circuit is shown in Fig. 3.25. The simulation current in the 6 Ω resistor is same as the calculated current.

Practice Problem 3.7 Use nodal analysis to calculate the node voltages for the circuit in Fig. 3.26. Use PSpice simulation to verify the results.

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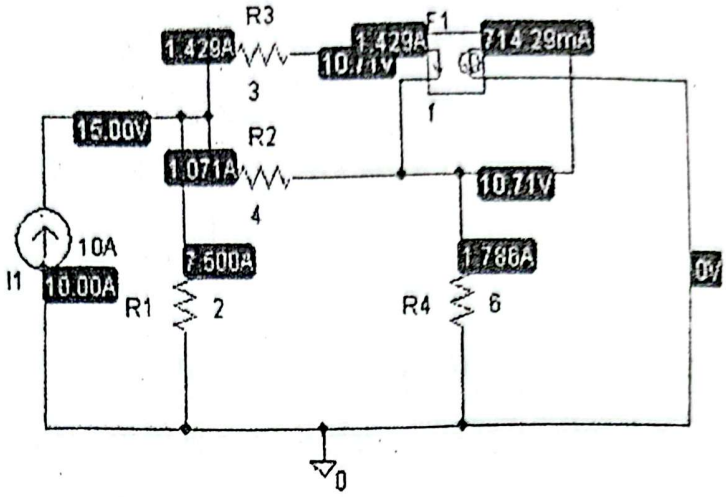
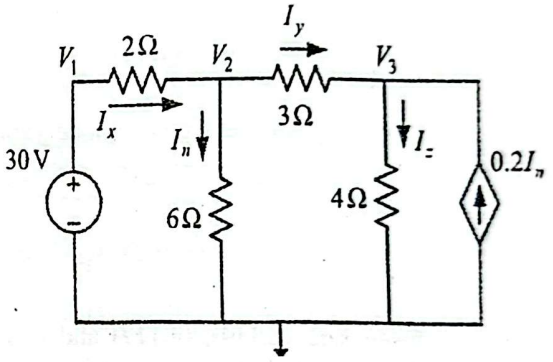


Fig. 3.25 PSpice simulation circuit for Example 3.7

Fig. 3.26 Circuit for Practice Problem 3.7



3.8 Supernode

Sometimes, in practical electrical circuits, a voltage source with or without series resistance is connected between the two non-reference (or one reference and one non-reference) nodes. When a voltage source is connected between two nodes, then it is known as a supernode. Both KVL and KCL are applied to the supernode to get simultaneous equations [1–5]. An electrical circuit with a supernode is shown in Fig. 3.27.

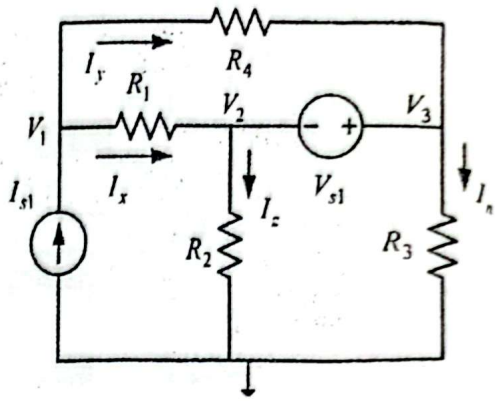
Applying KCL at node 1 (V_1) yields,

$$I_{s1} = I_x + I_y \tag{3.112}$$

$$I_{s1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_4} \tag{3.113}$$

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Fig. 3.27 An electrical circuit with supernode



$$(G_1 + G_4)V_1 - G_1V_2 - G_4V_3 = I_{s1} \tag{3.114}$$

Applying KCL at the supernode (the node with V_2, V_3 and V_{s1}) yields,

$$I_x = I_z + I_n \tag{3.115}$$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2} + \frac{V_3}{R_3} \tag{3.116}$$

$$G_1V_1 - (G_1 + G_2)V_2 - G_3V_3 = 0 \tag{3.117}$$

Applying KVL at the supernode yields,

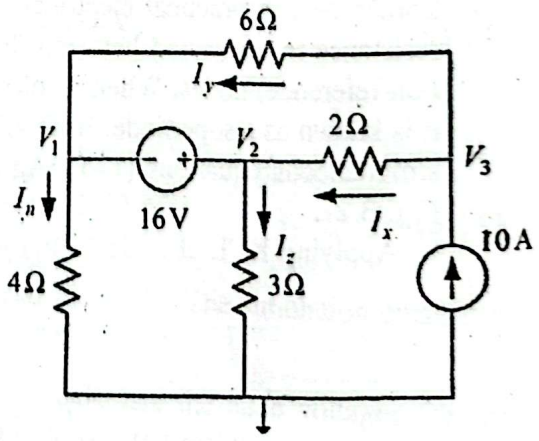
$$-V_2 - V_{s1} + V_3 = 0 \tag{3.118}$$

$$0V_1 - V_2 + V_3 = V_{s1} \tag{3.119}$$

From Eqs. (3.114), (3.117) and (3.119), the values of node voltages can be calculated if the other electrical quantities are given.

Example 3.8 An electrical circuit is shown in Fig. 3.28. Use nodal analysis to calculate the node voltages and the current in the 3 Ω resistor. Use PSpice simulation to determine the node voltages.

Fig. 3.28 Circuit for Example 3.8



Solution:

Applying KVL to the supernode yields,

$$-V_1 - 16 + V_2 = 0 \tag{3.120}$$

$$-V_1 + V_2 + 0V_3 = 16 \tag{3.121}$$

Applying KCL to the supernode yields,

$$I_x + I_y = I_n + I_z \tag{3.122}$$

$$\frac{V_3 - V_2}{2} + \frac{V_3 - V_1}{6} = \frac{V_1}{4} + \frac{V_2}{3} \tag{3.123}$$

$$6V_3 - 6V_2 + 2V_3 - 2V_1 - 3V_1 - 4V_2 = 0 \tag{3.124}$$

$$-5V_1 - 10V_2 + 8V_3 = 0 \tag{3.125}$$

Applying KCL to the node 3 yields,

$$10 = I_x + I_y \tag{3.126}$$

$$10 = \frac{V_3 - V_2}{2} + \frac{V_3 - V_1}{6} \tag{3.127}$$

$$60 = 3V_3 - 3V_2 + V_3 - V_1 \tag{3.128}$$

$$-V_1 - 3V_2 + 4V_3 = 60 \tag{3.129}$$

The value of the determinant of three simultaneous equations is,

$$D = \begin{vmatrix} -1 & 1 & 0 \\ -5 & -10 & 8 \\ -1 & -3 & 4 \end{vmatrix} = -1(-40+24) - 1(-20+8) = 28 \tag{3.130}$$

To evaluate V_2, D_2 parameter is calculated as,

$$D_2 = \begin{vmatrix} -1 & 16 & 0 \\ -5 & 0 & 8 \\ -1 & 60 & 4 \end{vmatrix} = -1(0-480) - 16(-20+8) = 672 \tag{3.131}$$

The value of the voltage at node 2 is determined as,

$$V_2 = \frac{D_2}{D} = \frac{672}{28} = 24 \text{ V} \tag{3.132}$$

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From Eqs. (3.121) and (3.125), the values of other node voltages can be determined as,

$$V_1 = 24 - 16 = 8 \text{ V} \tag{3.133}$$

$$V_3 = \frac{5 \times 8 + 10 \times 24}{8} = 35 \text{ V} \tag{3.134}$$

The value of the current in the 3 Ω resistor is,

$$I_{3\,\Omega} = \frac{V_2}{3} = \frac{24}{3} = 8 \text{ A} \tag{3.135}$$

The PSpice simulation circuit is shown in Fig. 3.29. The simulation current is found to be the same as the calculated current.

Practice Problem 3.8 Figure 3.30 shows an electrical circuit. Calculate the node voltages and the power absorbed by the 5 Ω resistor. Use PSpice simulation to determine the node voltages.

Example 3.9 Figure 3.31 shows an electrical circuit. Use nodal analysis to calculate the node voltages and the current in the 6 Ω resistor. Use PSpice simulation to determine the node voltages.

Solution:
The value of the voltage at node 1 is,

$$V_1 = 20 \text{ V} \tag{3.136}$$

Applying KVL at the supernode of the circuit in Fig. 3.32 yields,

$$-V_2 + 2I_x + V_3 = 0 \tag{3.137}$$

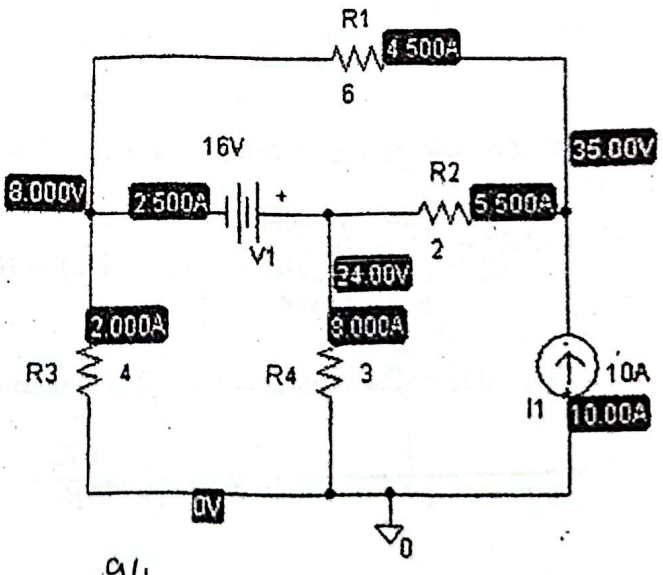


Fig. 3.29 PSpice simulation circuit for Example 3.8

Fig. 3.30 Circuit for Practice Problem 3.8

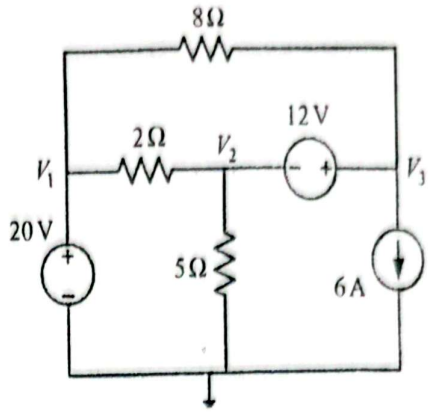


Fig. 3.31 An electrical circuit for Example 3.9

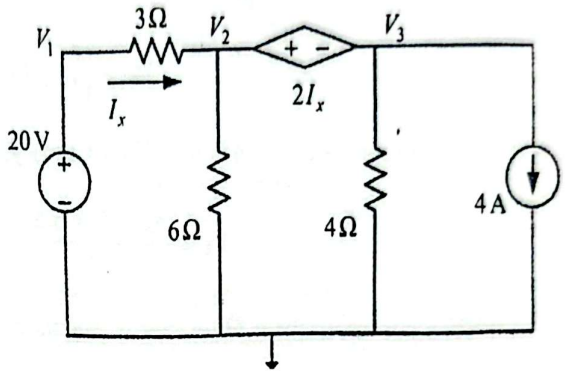
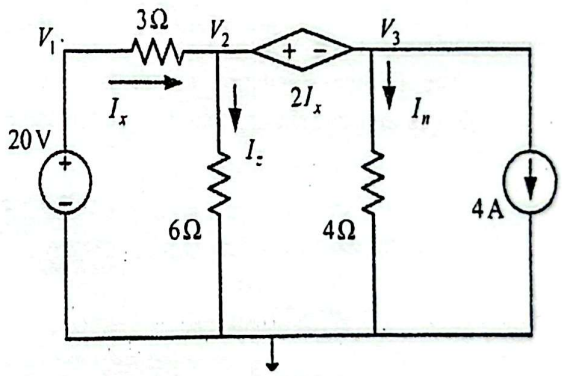


Fig. 3.32 An electrical circuit with directions of currents for Example 3.9



The current I_x through 3 Ω resistor can be written as,

$$I_x = \frac{20 - V_2}{3} \tag{3.138}$$

Substituting Eq. (3.138) into an Eq. (3.137) yields,

$$-V_2 + 2\frac{20 - V_2}{3} + V_3 = 0 \tag{3.139}$$

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$$-5V_2 + 3V_3 = -40 \tag{3.140}$$

Applying KCL at the supernode of Fig. 3.32 yields,

$$I_x = I_n + I_z + 4 \tag{3.141}$$

$$\frac{20 - V_2}{3} = \frac{V_2}{6} + \frac{V_3}{4} + 4 \tag{3.142}$$

$$80 - 4V_2 = 2V_2 + 3V_3 + 48 \tag{3.143}$$

$$6V_2 + 3V_3 = 32 \tag{3.144}$$

From Eqs. (3.140) and (3.144), the value of the node voltage can be determined as,

$$V_2 = \frac{72}{11} = 6.54 \text{ V} \tag{3.145}$$

Substituting Eq. (3.145) into Eq. (3.144) yields,

$$V_3 = \frac{32 - 6 \times 6.54}{3} = -2.41 \text{ V} \tag{3.146}$$

The current in the 6 Ω resistor is calculated as,

$$I_{6\,\Omega} = \frac{V_2}{6} = \frac{6.54}{6} = 1.09 \text{ A} \tag{3.147}$$

The PSpice simulation circuit is shown in Fig. 3.33. The simulation node voltages and current are found to be the same as the calculated results.

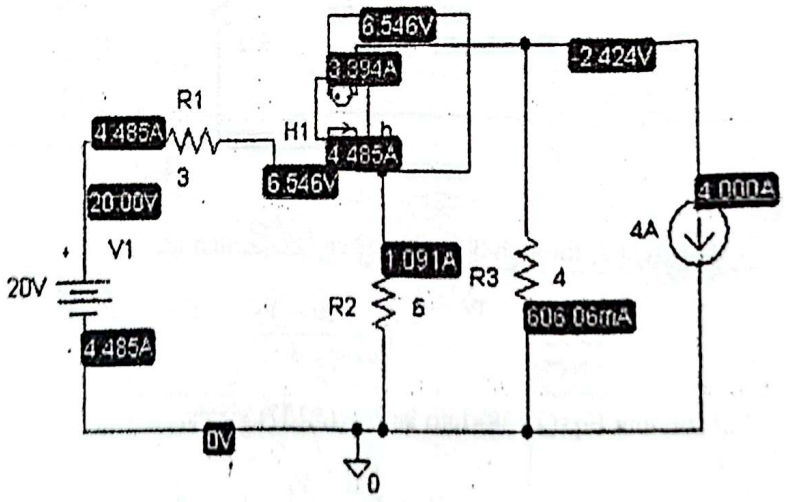


Fig. 3.33 PSpice simulation circuit for Example 3.9

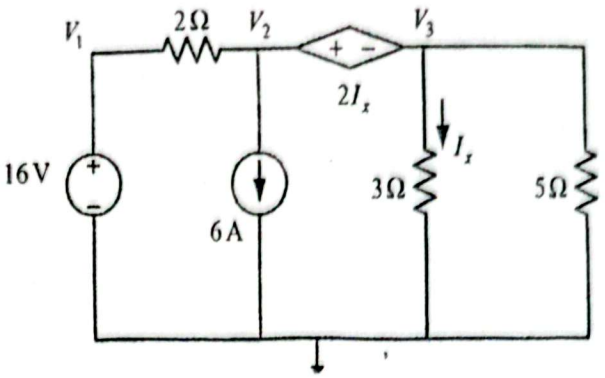


Fig. 3.34 Circuit for Practice Problem 3.9

Practice Problem 3.9 An electrical circuit is shown Fig. 3.34. Determine the node voltages and the current in the 5 Ω resistor using nodal analysis. Use PSpice simulation to determine the node voltages.

Exercise Problems

3.1 Three simultaneous equations of voltages of a multisource electrical circuit are given by $2V_1 + 4V_2 - 5V_3 = 8$, $V_1 - 2V_2 + 3V_3 = 6$, $2V_1 + 3V_2 + 4V_3 = 8$. Determine the unknown voltages.

3.2 Calculate the current in the 3 Ω resistor of the circuit in Fig. 3.35 using mesh analysis. Use PSpice simulation to verify the result.

3.3 Use mesh analysis to determine the current in the 4 Ω resistor of the circuit in Fig. 3.36 and verify the result by PSpice simulation.

3.4 A multisource electrical circuit is shown in Fig. 3.37. Determine the current in the 1 Ω resistor using mesh analysis. Verify the result by PSpice simulation.

3.5 An electrical circuit with multisource is shown in Fig. 3.38. Use mesh analysis to calculate the current in the 4 Ω resistor and verify the result by PSpice simulation.

3.6 Fig. 3.39 shows a multisource electrical circuit. Calculate the current in the 5 Ω resistor using mesh analysis and verify the result by PSpice simulation.

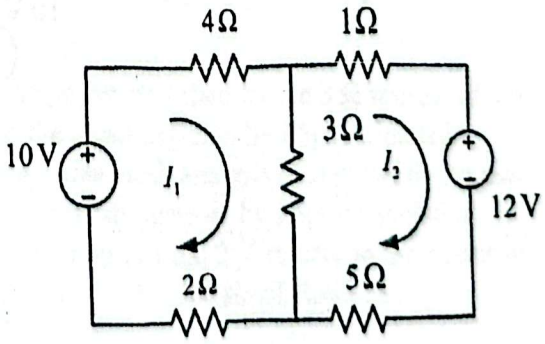


Fig. 3.35 Circuit for exercise problem 3.2

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Fig. 3.36 Circuit for exercise problem 3.3

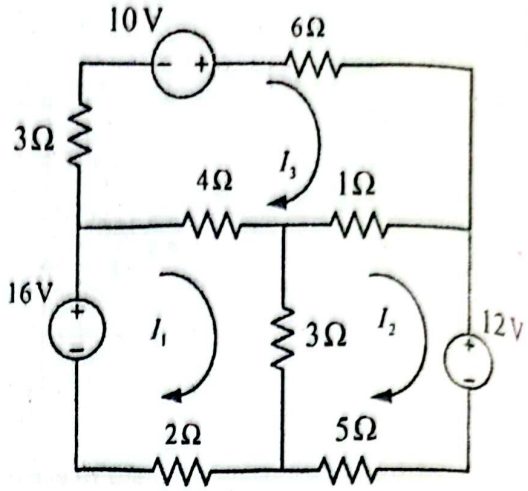


Fig. 3.37 Circuit for exercise problem 3.4

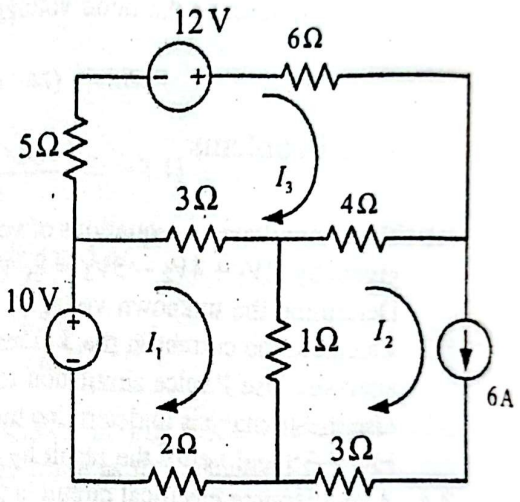
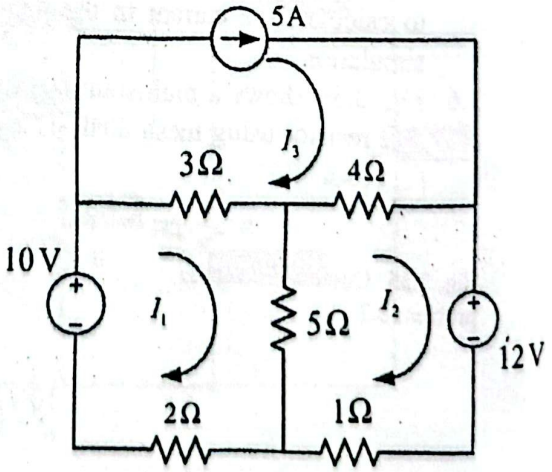


Fig. 3.38 Circuit for exercise problem 3.5



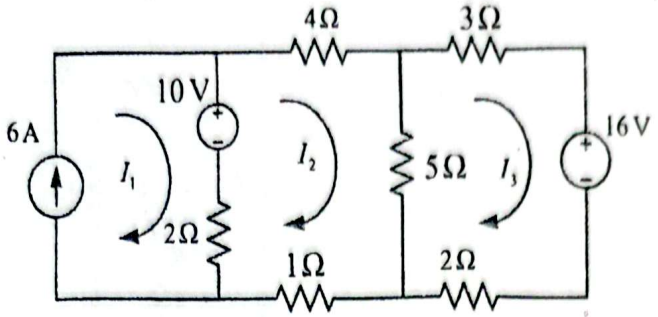


Fig. 3.39 Circuit for exercise problem 3.6

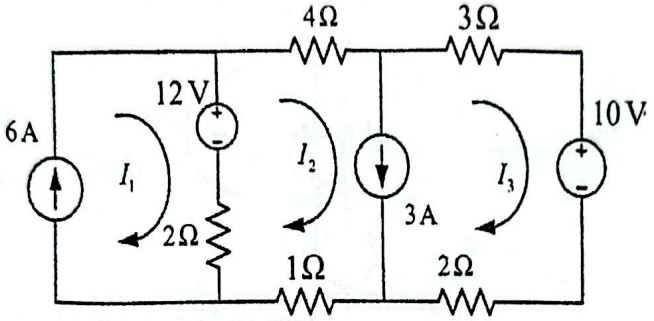
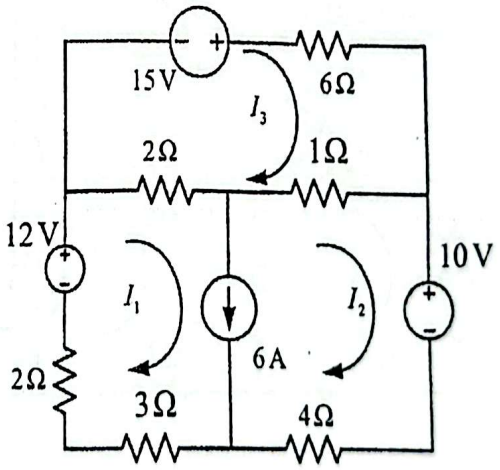


Fig. 3.40 Circuit for exercise problem 3.7

Fig. 3.41 Circuit for exercise problem 3.8



3.7 Use mesh analysis to calculate the power absorbed by the 3 Ω resistor of the circuit in Fig. 3.40 and calculate the mesh currents by PSpice simulation.

3.8 Fig. 3.41 shows an electrical circuit. Use mesh analysis to calculate the current in the 1 Ω resistor and calculate the mesh currents by PSpice simulation.

3.9 Use mesh analysis to calculate the current in the 2 Ω resistor in the circuit in Fig. 3.42. Calculate the mesh currents by PSpice simulation.

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Fig. 3.42 Circuit for exercise problem 3.9

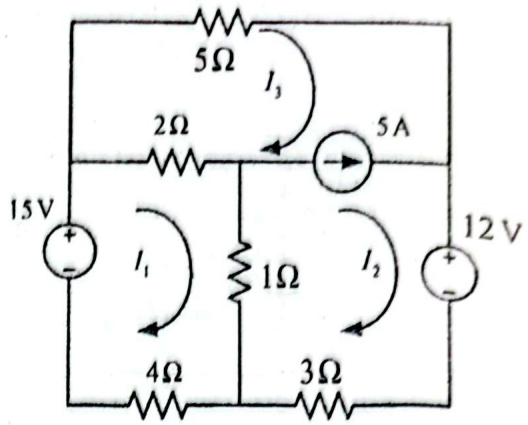


Fig. 3.43 Circuit for exercise problem 3.10

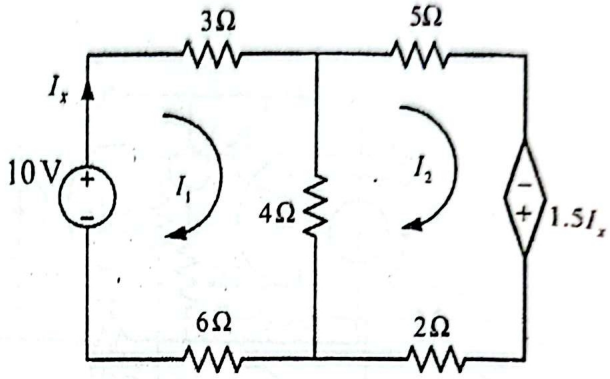
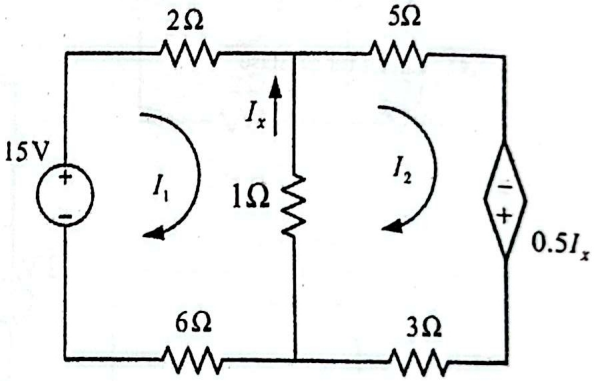


Fig. 3.44 Circuit for exercise problem 3.11



3.10 An electrical circuit is shown in Fig. 3.43. Use mesh analysis to calculate the current in the 4 Ω resistor. Find the mesh currents by PSpice simulation.

3.11 Fig. 3.44 shows an electrical circuit. Determine the current in the 1 Ω resistor using mesh analysis. Find the mesh currents by PSpice simulation.

3.12 Use mesh analysis to determine the current in the 3 Ω resistor of the circuit in Fig. 3.45. Find the mesh currents by PSpice simulation.

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Fig. 3.45 Circuit for exercise problem 3.12

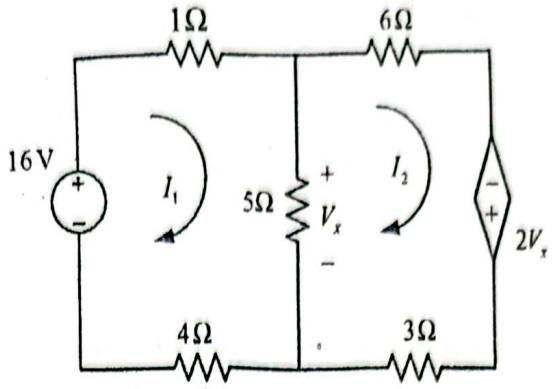


Fig. 3.46 Circuit for exercise problem 3.13

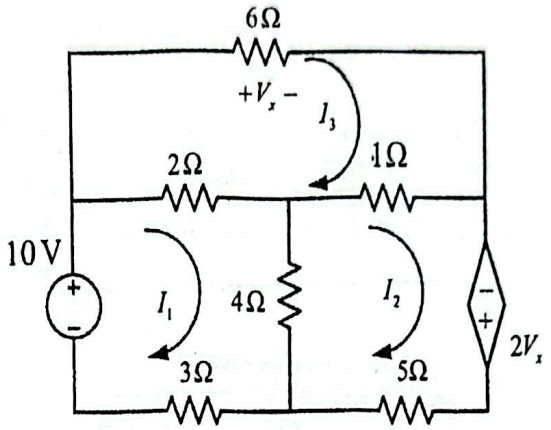
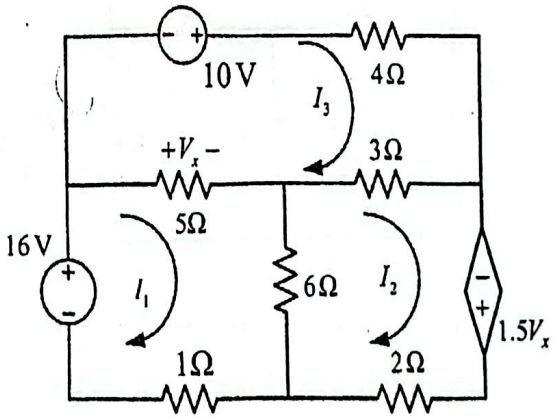


Fig. 3.47 Circuit for exercise problem 3.14



3.13 Fig. 3.46 shows a circuit. Use mesh analysis to determine the current in the 2 Ω resistor and find the mesh currents by PSpice simulation.

3.14 Use mesh analysis to determine the current in the 2 Ω resistor of the circuit in Fig. 3.47.

3.15 Fig. 3.48 shows an electrical circuit. Determine the mesh currents and verify the results by PSpice simulation.

Fig. 3.48 Circuit for exercise problem 3.15

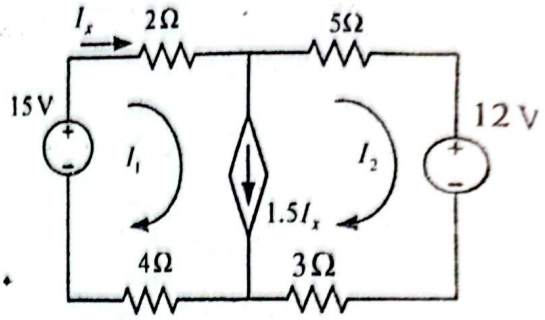


Fig. 3.49 Circuit for exercise problem 3.16

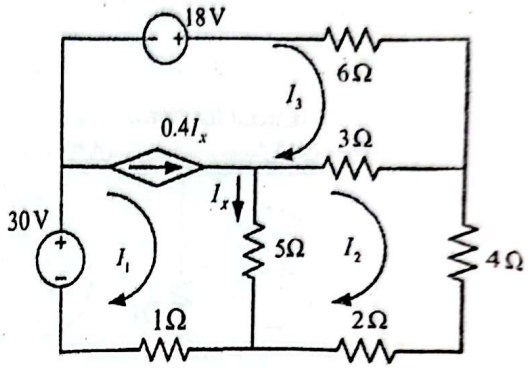
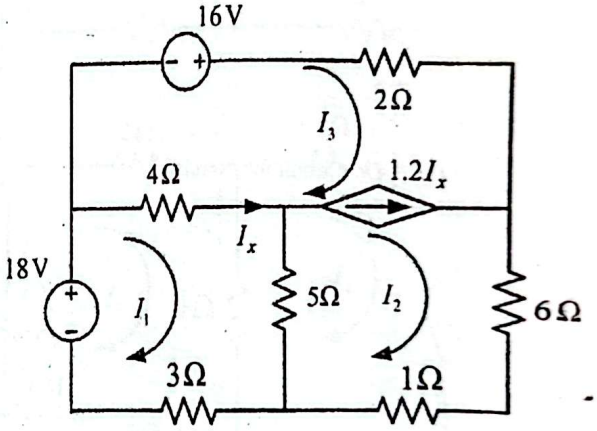


Fig. 3.50 Circuit for exercise problem 3.17



3.16 An electrical circuit is shown in Fig. 3.49. Use mesh analysis to find the current in the 3 Ω resistor. Find the mesh currents by PSpice simulation.

3.17 Use mesh analysis to find the current in the 5 Ω resistor of the circuit in Fig. 3.50. Find the mesh currents by PSpice simulation.

3.18 Use nodal analysis to find the current in the 6 Ω resistor of the circuit in Fig. 3.51. Determine the node voltages by PSpice simulation.

3.19 An electrical circuit is shown in Fig. 3.52. Use nodal analysis to find the current in the 6 Ω resistor and find the node voltages by PSpice simulation.

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Fig. 3.51 Circuit for exercise problem 3.18

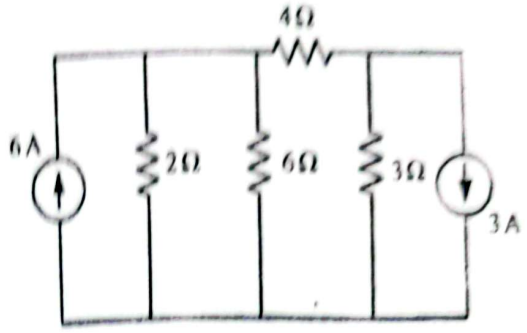


Fig. 3.52 Circuit for exercise problem 3.19

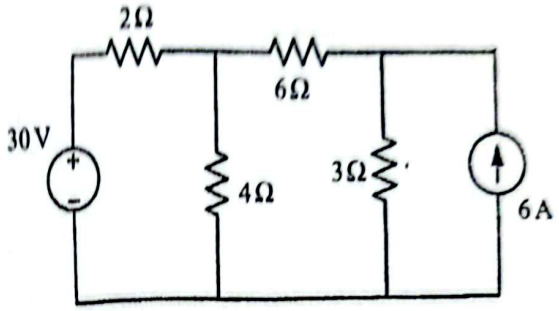
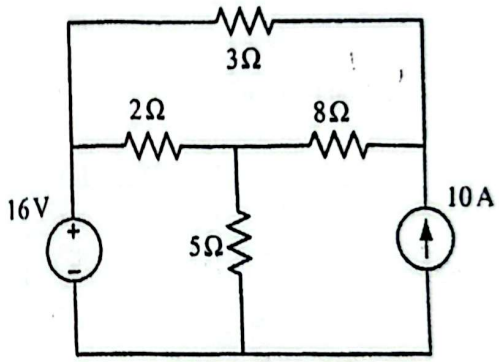


Fig. 3.53 Circuit for exercise problem 3.20



3.20 Fig. 3.53 shows an electrical circuit. Find the current in the 5 Ω resistor using nodal analysis and verify the result by PSpice simulation.

3.21 An electrical circuit is shown in Fig. 3.54. Calculate the current in the 2 Ω resistor using nodal analysis and verify the result by PSpice simulation.

3.22 Use nodal analysis to calculate the current in the 3 Ω resistor of the circuit in Fig. 3.55. Verify the result by PSpice simulation.

3.23 An electrical circuit is shown in Fig. 3.56. Use nodal analysis to calculate the current in the 5 Ω resistor. Verify the result by PSpice simulation.

3.24 Fig. 3.57 shows an electrical circuit. Use nodal analysis to calculate the current in the 3 Ω resistor. Verify the result by PSpice simulation.

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Fig. 3.54 Circuit for exercise problem 3.21

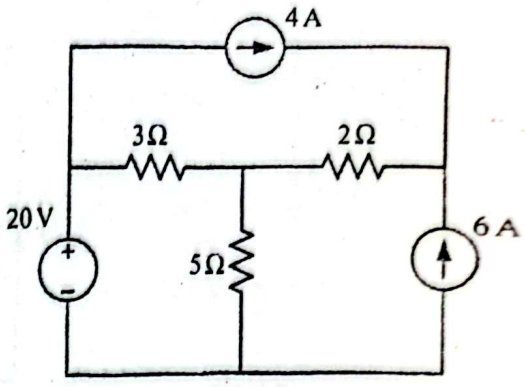


Fig. 3.55 Circuit for exercise problem 3.22

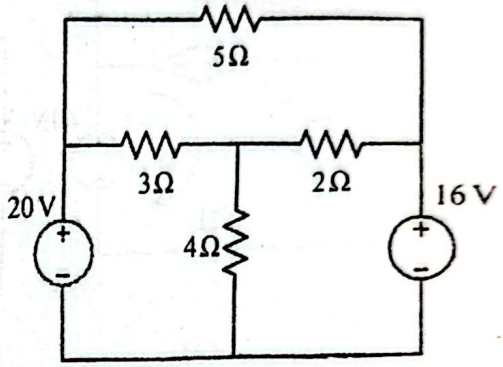


Fig. 3.56 Circuit for exercise problem 3.23

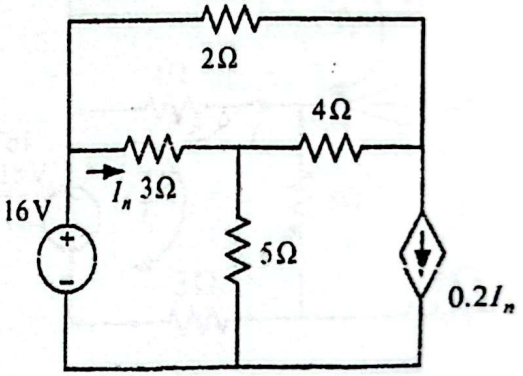
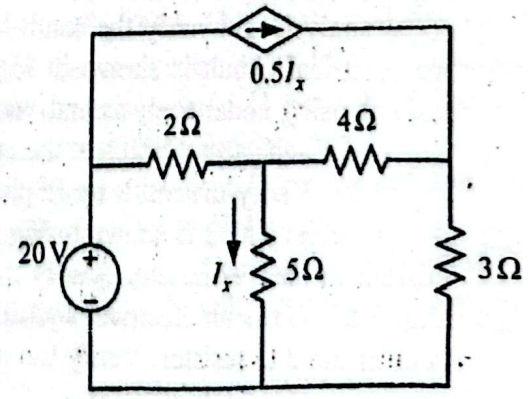


Fig. 3.57 Circuit for exercise problem 3.24



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Fig. 3.58 Circuit for exercise problem 3.25

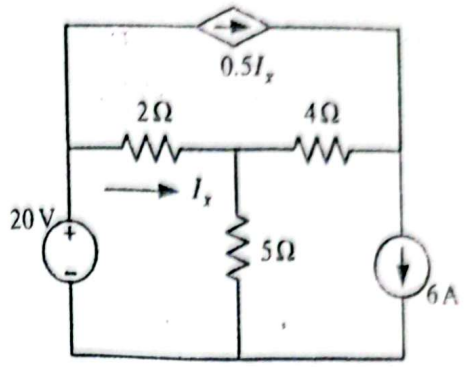


Fig. 3.59 Circuit for exercise problem 3.26

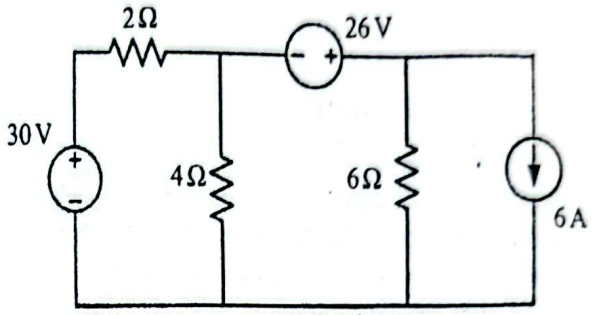
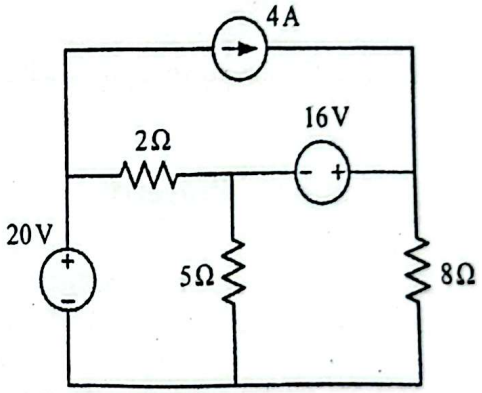


Fig. 3.60 Circuit for exercise problem 3.27



3.25 Fig. 3.58 shows an electrical circuit. Use nodal analysis to calculate the current in the 4 Ω resistor. Verify the result by PSpice simulation.

3.26 Use nodal analysis to calculate the current in the 4 Ω resistor of the circuit in Fig. 3.59. Verify the result by PSpice simulation.

3.27 Fig. 3.60 shows an electrical circuit. Determine the current in the 5 Ω resistor using nodal analysis and verify the result by PSpice simulation.

3.28 An electrical circuit is shown in Fig. 3.61. Calculate the current in the 2 Ω resistor using nodal analysis and verify the result by PSpice simulation.

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